UNIVERSITY OF JORDAN

FACULTY OF GRADUATE STUDIES

COMPUTATION AND LAYOUT OF SPIRAL CURVES BY ELECTRONIC MEANS AND COMPUTER PROGRAMMING

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Submitted in partial fulfillment of the requirements for the degree of master of science in civil engineering / Highways and Traffic

Faculty of Graduate Studies

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October , 1993

This thesis was defended successfuly on October 27, 1993

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ACKNOWLEDGEMENTS

I wish to express my sincere thanks to Professor Youssuf Siyam, for initiating my interest in the subject of this thesis, and for the continuous encouragement that I have received from him. Special gratitude is due to Mr. Salem J. Faoury, for his valuable help to me throughout my work. Iam also indebted to my wife for her moral support and constant assistance.

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ABSTRACT

Computation And Layout Of Spiral Curves By Electronic Means And Computer Programming

In order to set out horizontal curves in the field using the Total Station Instrument (TSI), mathematical models have been developed for the design of simple circular, compound, reverse and spiral curves. special emphasis has been placed on the design of the differents types of spiral curves such as double spiral, spiraled compound and spiraled reverse curves.

Using these mathematical models, a major computer program has been written to carry out the following tasks:

1- Conduct a traverse adjustment for the lines that link between a series control stations points of intersections.

2- Compute elements of any curve.

3- Compute absolute coordinates for a group of points on each curve.

4- Compute the azimuth and length of the line that joins the first point on the curve to any other point along the same curve.

It is expected that this technique will serve to set out these types of horizontal curves in an accurate, quick and efficient way.

CHAPTER 1

INTRODUCTION

1-1 GENERAL

Horizontal curves are traditionally set out by deflection angles as measured by theodolite, and chords as measured by tapes. This method does not give a high degree of precision due to many reasons among which :

1- chords are used instead of arcs, although computations are based on arcs lengths.

2- Errors in taping

3- Inaccurate theodolite readings and settings.

Moreover, the whole horizontal curve cannot be set out from just a single instrument position. The instrument has to be moved several times depending on topography and type of curve. For example, if we have an equal-tangent spiraled circular curve, then the theodolite has to be set up over 3 points (TS,SC and ST). This means that more time will be needed in order to set out all the points of curve.

With the rapid growth and progress in the development of computers and electronic surveying instruments, new techniques have to be developed for the design and setting out of horizontal curves. Such new techniques will help to accomplish the surveying work, required in the design and setting out process of horizontal curves, in an easy, quick and efficient manner.

As an example for such new techniques, if the coordinates of all points on the horizontal curve are determined, then it is possible to locate all these points from just a single instrument position, in an efficient and accurate manner, using the Total Station Instrument (TSI)

When the TSI is used, then all the curve points can be fixed from one instrument position, that can be the point of intersection PI, or any other point of known coordinates, whether this point is situated on the curve or not. After orienting on a given control point whose coordinates are known, it is only needed to feed the TSI with the computed absolute coordinates of all curve points, then the reflector has to be moved about until the coordinates of the required point are obtained.

1-2 OBJECTIVE OF THE RESEARCH

The objective of this research is to develop procedures and advanced surveying computer programs in the subject of design and setting out of simple circular, compound, reverse and spiral curves. These newly developed procedures and programs will conform to the conditions and potentials of the country, and accordingly this work will contribute in finding a substitute for some of the market surveying programs that are bought from foreign countries.

1-3 SCOPE OF INVESTIGATION

1- In order to develop computer programs in the subject of design and setting out of horizontal curves, mathematical models have to be developed. Such mathematical models present the method of computing the absolute coordinates of points on the horizontal curves. Special concentration will be given to the different types of spiral curves. Mathematical models will be developed for design of the following types of horizontal curves:

   a- Simple Circular curves.

   b- Spiraled Circular curves.

   c- Double Spiral curves.

   d- Compound curves.

   e- Spiraled Compound curves.

   f- Simple Reverse curves.

   g- Spiraled Reverse curves.

2- The main purpose of this research is to turn the above mentioned mathematical models into a major computer program that is capable to perform the following:

   a- Compute the absolute coordinates for all points on any type of horizontal curve, thus making it possible to set out such curves in the field by the TSI.

b - Compute the curve elements such as tangent length, curve length, external distance, shift,....ets.

c- Carry out an entire traverse adjustment process, if necessary, in order to determine the coordinates of the points of intersections (PIs). This will be needed if the PI coordinates are not furnished as in the case of connecting traverse. The design process of the curve can not proceed if the PI coordinates are not given.

d- As an additional feature, the computer program will include an output in the form of a table that shows the distance and azimuth of the line joining the the first point on curve to any other point along the curve. Such table will be useful for checking purposes.

CHAPTER 2

LITERATURE SURVEY

2-1 Introduction

The transition (Spiral) curves are introduced between tangent and circular curve or between two circular arcs of different radii, in such a way that the change of curvature will be brought about in a gradual way. These spiral curves have been found necessary on high-speed railroads from the standpoint of comfortable operation and of gradually bringing about the full superelevation of the outer edge on curve [1,2].

When a moving vehicle traveling on a straight portion of a highway enters a circular curve, a full centrifugal force developes at once which tends to overturn the moving vehicle. The sudden change of curvature from zero on the tangent to a certain value at the beginning of the circular curve will cause discomfort to the passengers and results in jolting the moving vehicle.

In traveling from a straight line through a circular arc the steering change can not be obtained without a transition. This transition can be effected within the limit of normal lane width in the case where the curvature is not sharp and the speed is low. In such situation there will be no need for spiral. However, the absence of transition curves on high design speed roadways that have sharp curves, will force the driver to reduce his speed and, will possibly result in encroachment on the adjoining lanes which mandate an extra lane width to keep the vehicle in their lanes [3,4].

When a spiral is inserted between a straight and circular arc the curvature and centrifugal force will vary linearly from zero at the beginning of the spiral, to a fixed value at it's junction with the circular arc [1]. To maintain smooth driving, it will be thus necessary to insert a transition curve between the straight portion and the circular arc or between the two arcs of different radii

The insertion of the spiral will provide a road that appears continuous without breaks. Such breaks may prevent the driver from looking beyond them and therefore forcing him to reduce his speed. In addition the spiral can be easily adapted to the contours of the ground especially in mountainous terrain and such better adaptation to the terrain results in less excavation and consequently less construction cost [5].

The use of spiral curves will provide a convenient way to bring about the necessary change in cross slope from the normal crowned section on the straight to the fully superelevated section on the circular curve. This change in cross slope will be effected on the entire length of spiral in a manner closely fitting the speed-radius relationship of the moving vehicle [1,3].

In conclusion, the introduction of spiral curves in horizontal alinement offers th following advantages :

1- It eliminates the discomforts caused to the passengers by sudden change of curvature.
2- It reduces overturning and side slipping of moving vehicles that is caused by the sudden change in centrifugal force.
3- It minimizes encroachment on adjoining lanes and enhances uniformity in speed.
4- It facilitates the introduction of superelevation in proportion to the rate of change in curvature.
5- It facilitates the transition in width when pavement widening is to be applied around circular curve.
6- It enhances appearance of the highway.

2-2 TYPES OF TRANSITION CURVES

1- Cubic parabola: This is the most widely used in practice because of its simplicity. The basic equation of the cubic parabola is of the form (see Fig. 2-1) [1]:

$$Y= \frac{X^3}{6\ R\ L_s} \qquad (2.1)$$

Where

X,Y: coordinates of points on the cubic parabola.

R : radius of circular curve (meters).

L_s : length of the spiral curve .

2- The Lemniscate: Its basic equation in polar coordinates is of the form (see Fig. 2-2):

$$P^2 = K^2 \sin 2W \qquad (2.2)$$

Where

P : the polar radius

W : the polar angle

K : constant

This type of transition curves is used mainly in rugged terrain where it is difficult to fix the points using the ordinary coordinates [1].

3- The Clothoid (Spiral): This type is most commonly used and preferred because of its geometric properties. The curvature of the clothoid varies linearly from zero at the tangent end of the clothoid to the degree of curve of the circular arc at the circular curve end (see Fig. 2-3).

The basic equation of the clothoid is

$$C = L_s * R \qquad (2.3)$$

Where

C : constant

R : radius of circular curve

L_s: the clothoid (spiral) length [1]

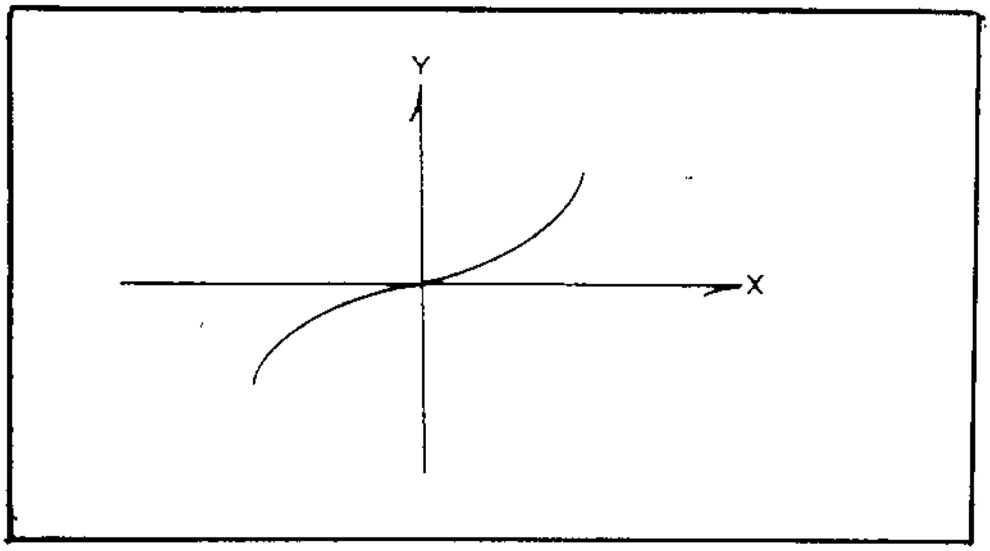


Figure (2-1): Cubic parabola [1]

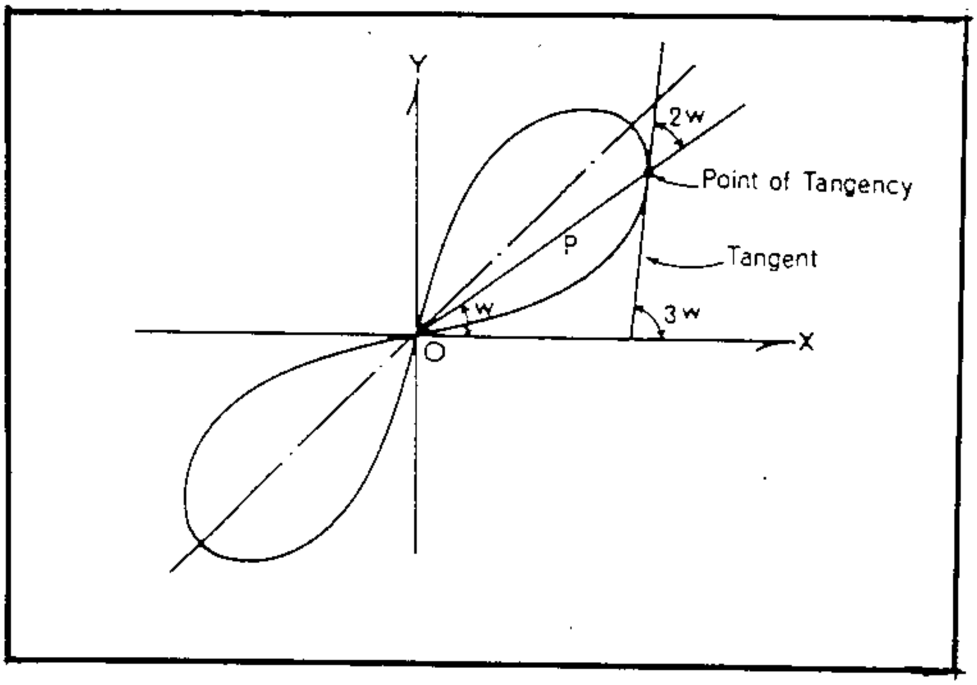


Figure (2-2): Lemniscate[1]

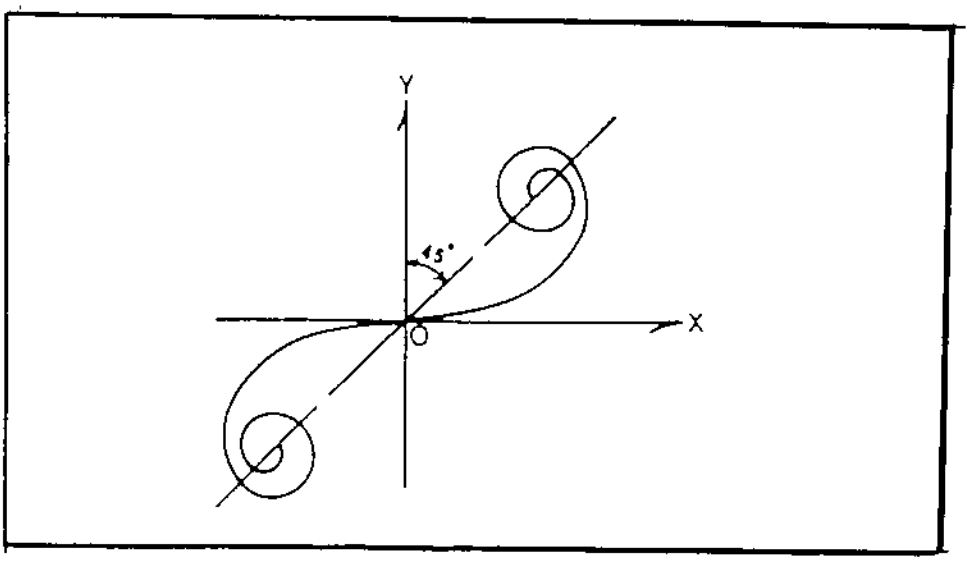


Figure (2-3): Clothoide [1]

2-3 SUPERELEVATION

When a vehicle moves from a straight to a curve, it is subjected to an outward radial (centrifugal) force. This force is counterbalanced by the vehicle weight component related to the roadway superelevation, and by the side frictional force between the tyres and road surface. The superelevation is attained by raising the outer edge of the road pavement above the inner edge. From the laws of mechanics it can be shown that

$$e = \frac{V^2}{127.12*R} - f_s \qquad (2.4)$$

Where

e : Superelevation rate

V : Design speed (km/hr)

R : Radius of curve (m)

f_s: Coefficient of side friction.

If the width of roadway (b) is given, then the amount of superelevation (H) is given by

$$H = b\ e \qquad (2.5)$$

The transition curve is inserted between the straight and circular curve so that the superelevation is applied gradually from zero at the point of commencement of the transition curve (TS) to its full value at the junction of the transition curve and circular curve (SC). The maximum value of superelevation rate that can be used in highway design is 0.12. Superelevation value of 0.08 is generally desirable [1,3].

2-4 THE SPIRAL LENGTH

Several methods are used for determination of spiral length among which are the following :

1- By using an arbitrary length of say 50 or 100 or 150m, based on previous experience and aesthetic appearance[1]

2- By using a maximum relative profile gradient of superelevation (e.g 1/250,1/200,...etc) along the spiral length [1,3,6].

Let H be the amount of superelevation in centimeters.

$\frac{1}{S}$ be the relative profile gradient superelevation.

L_s be the spiral length.

then

$$L_s = \frac{S * H}{100} \qquad (2.6)$$

3- By using an arbitrary rate of superelevation per unit of time [1,6].

Let V be the design speed in km/hr.

H be the amount of superelevation in cm.

L_s be the spiral length.

A be the time rate (A varies from 2.5 - 5 cm/sec)

then

H = time rate * time taken to travel distance L_s

$$H = A * \frac{L_s}{V}$$

$$L_s = \frac{H * V}{A} \qquad (2.7)$$

4- The minimum length of spiral required is determined on the basis of a limited rate of change in radial acceleration.

The radial acceleration increases from zero at the start of spiral to V^2/R at the joint with circular arc,

therefore

$$\text{Rate of change in radial acceleration} = a = \frac{V^2/R}{t}$$

$$= \frac{V^2/R}{L_s/V}$$

$$= \frac{V^3}{R\,L_s}$$

Thus the length of the spiral curve is given by this equation:

$$L_s = \frac{V^3}{R\,a} \qquad (2.8)$$

where

R :radius of circular curve (m).

V :design speed (km/hr)

The factor a is an empirical value indicating the comfort and safety. Values ranging from 0.3 to 0.9 m/sec^3 have been used for highway design and a value of 0.3 m/sec^3 for railroad operation[1,7].

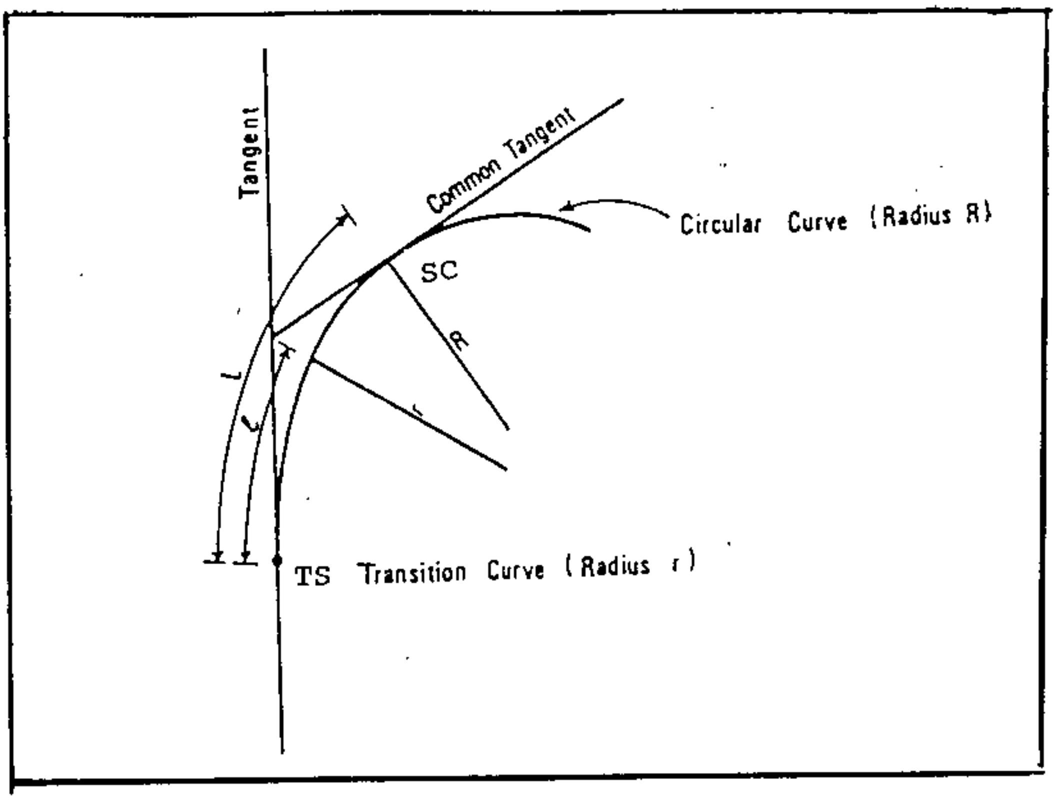


Figure (2-4): Simple Spiral(the basic formula lr=LR)

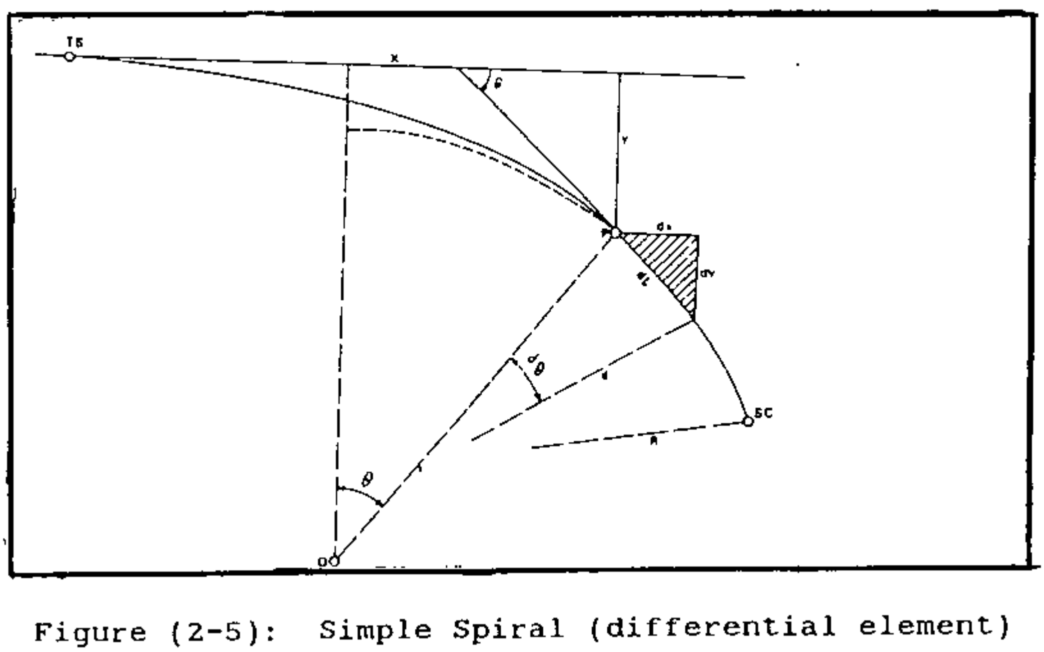


Figure (2-5): Simple Spiral (differential element)

2-5 SIMPLE SPIRAL

The basic law of simple spiral is that the radius at any point on the spiral is inversely proportional to the distance measured along it from the point of change from tangent to spiral curve (TS). Since there is an inverse relation between radius and degree of curve ,it can be stated that the degree of curve of the spiral increases at a uniform rate from zero at the TS to the degree D_c of the circular arc at the point of change from spiral to circular curve (SC) [1,2].

Referring to Fig. 2-4, one can write [1]

$$L_s * R = l\ r = K \qquad (2.9)$$

Where

L_s: Total spiral length

R : Radius of circular curve

K : Constant

considering a differential element of the spiral (Fig. 2-5) then

$$dl = r\ d\theta$$

$$d\theta = \frac{dl}{r} = \frac{l\ dl}{R\ L_s}$$

By integration we obtain,

$$\theta = \frac{l^2}{2\ R\ L_s} \qquad (2.10)$$

at point SC where r = R and $l = L_s$ the maximum value of

θ that is called the spiral central angle is obtained and is denoted by ϕ.

$$\phi = \frac{L_s}{2R} \qquad (2.11)$$

or

$$\phi = \frac{L_s * D_c}{60} \qquad (2.12)$$

Where

D_c is the degree of curve of the circular arc based on arc definition of 30 m or 100 ft..

When equation (2.10) is divided by equation (2.11) and the resulting equation is solved for θ ,it will be found that:

$$\theta = \left(\frac{1}{L_s}\right)^2 * \phi \qquad (2.13)$$

Which means that the spiral angle at any point on the spiral curve is directly proportional to the square of the length ratio measured from the **TS** .

For a differential element of length dl and a central angle of $d\theta$, the following equations can be written (Fig. 2-5):

$$dx = dl \cos \theta$$

$$dy = dl \sin \theta$$

Expressing the sine and cosine as power series, then we have

$$dx = dl \left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \text{negligible terms} \right)$$

$$dy = dl \left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \text{negligible terms} \right)$$

By substituting the value of θ from eq. 2.13 in the the above equations we get

$$dx = dl\ (1 - \frac{\phi^2\ l^4}{2\ L_s^4} + \frac{\phi^4\ l^8}{24\ L_s^8} - \text{negligible terms })$$

By integration we get

$$x = l - \frac{\phi^2\ l^5}{10\ L_s^4} + \frac{\phi^4\ l^9}{216\ L_s^8} + \text{negligible terms })$$

If we substitute the value of $\phi = (\frac{L_s}{l})^2 * \theta$ from eq. 2.13 into the above equation then we get

$$x = l\ (\ 1 - \frac{\theta^2}{10} + \frac{\theta^4}{216} - \ldots\ldots.) \qquad (2.14)$$

Using a similar procedure we obtain

$$y = l\ (\frac{\theta}{3} - \frac{\theta^3}{42} + \frac{\theta^5}{1320} - \ldots\ldots..) \qquad (2.15)$$

The above two equations give the local coordinates of any point along the spiral curve at a distance l from the TS station .Note that x is measured along the tangent starting from TS ,and y is the offset from the tangent. The angle ϕ in equations (2.14) and (2.15) is measured in radians. When it is required to compute the local coordinates of point SC then we substitute L_s for l and ϕ for θ in equations (2.14) and (2.15), thus we get:

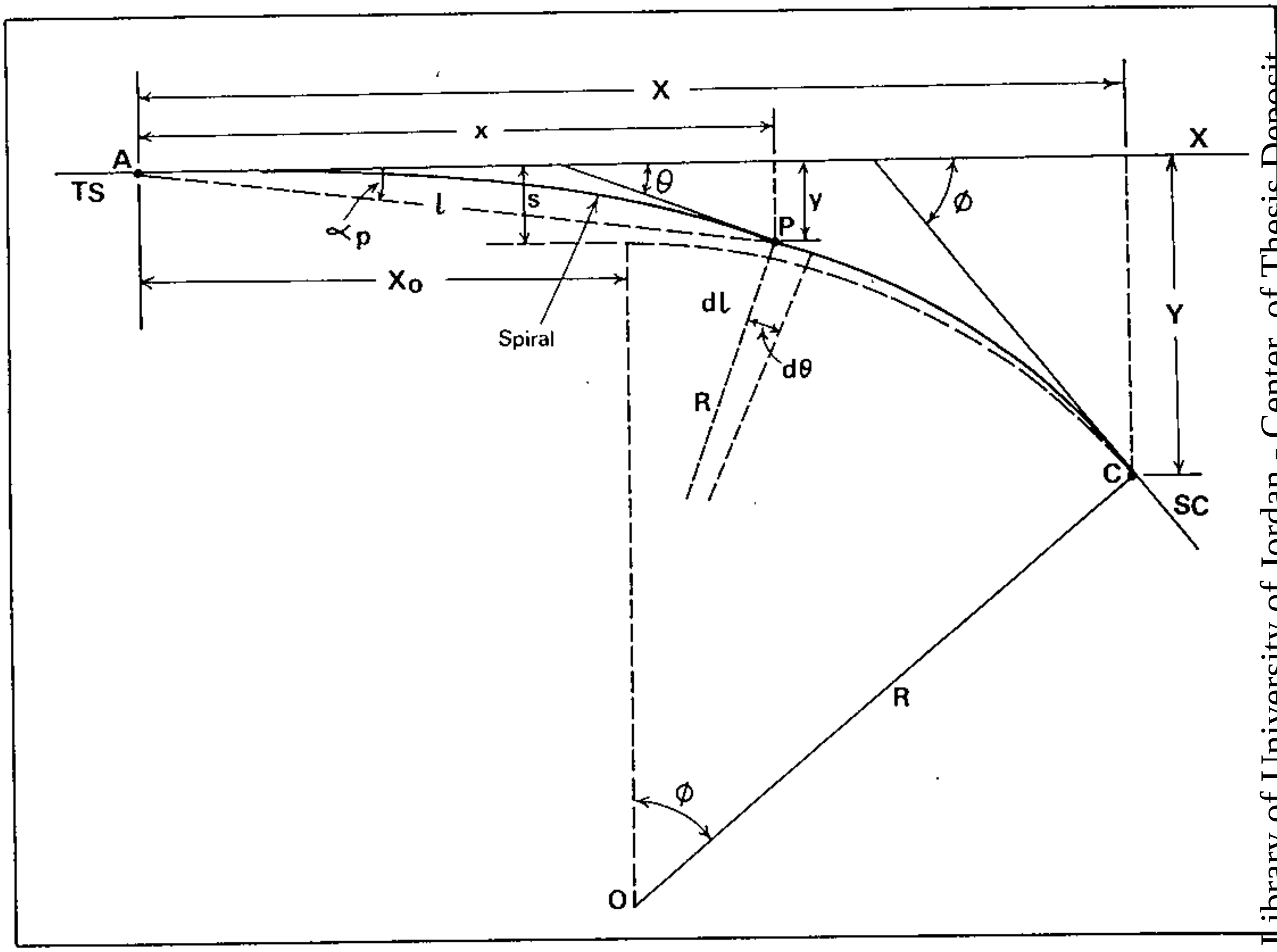


Figure (2-6): Basic Elements of Simple Spiral [2]

$$X = L_s \left(1 - \frac{\phi^2}{10} + \frac{\phi^4}{216} - \ldots\ldots\ldots\ldots\right) \qquad (2.16)$$

$$Y = L_s \left(\frac{\phi}{3} - \frac{\phi^3}{42} + \frac{\phi^5}{1320} - \ldots\ldots\ldots\right) \qquad (2.17)$$

From Fig. 2-6, it can be seen that

$$X_o = X - R * \sin \phi \qquad (2.18)$$

$$S = Y - R * (1 - \cos \phi) \qquad (2.19)$$

Where

X_o : the distance measured along the tangent from the TS to the unshifted point of curvature (PC).

S : the shift through which the circular arc must be shifted inward to make room for the insertion of the spiral [1,2]

2-6 SPIRALED CIRCULAR CURVES

2-6-1 Equal-tangent spiraled circular curve (symmetrical)

Referring to Fig. 2-7, the following symbols are used:

L_s : spiral length .

TS : point of change from back tangent to left spiral.

SC : point of change from left spiral to circular curve.

CS : point of change from circular to right spiral curve.

ST : point of change from right spiral to forward tangent

O : center of circular curve

Es : external distance

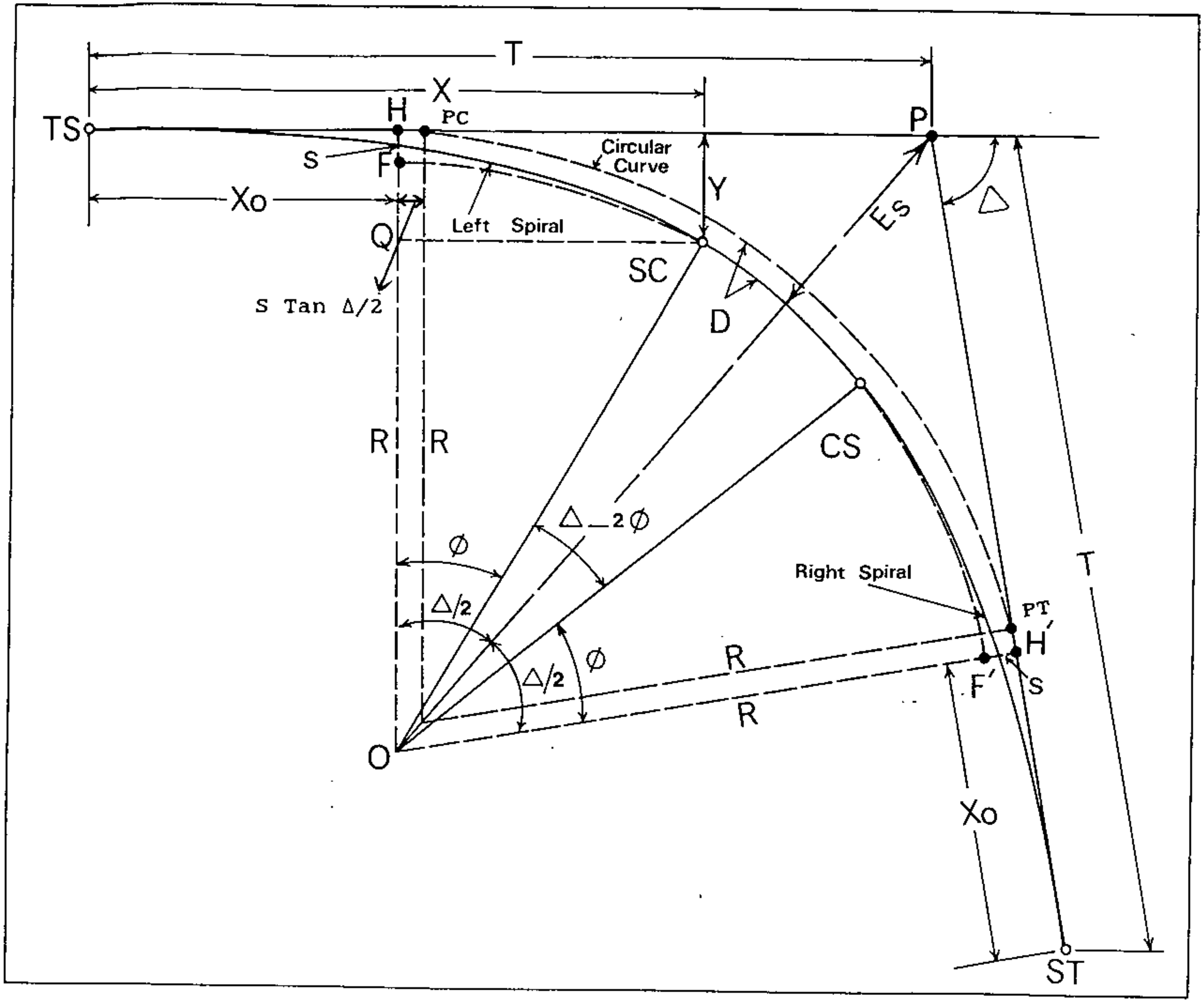


Figure (2-7): Equal-Tangent Spiraled Circular Curve

S : the shift of the original circular curve due to the insertion of the spirals

X_o : the distance between points TS and the PC of the unshifted circular curve

X&Y: local coordinates of point SC relative to point TS. The X-coordinate is measured along the tangent and the Y-coordinate is perpendicular to the tangent

ϕ : spiral central angle, $\phi = L_s/2R$

R : radius of circular curve

T : tangent length [1]

In this case the left and right spirals connected to the circular curve are of equal length . This will result in a symmetrical curve with equal tangents (Fig. 2-7).

The tangent distance is computed using the following equation:

$$T = (R + S) \tan \frac{\Delta}{2} + X_o \qquad (2.20)$$

Where

$$X_o = X - R * \sin \phi \qquad (2.18)$$

$$S = Y - R * (1 - \cos \phi) \qquad (2.19)$$

$$X = L_s \left(1 - \frac{\phi^2}{10} + \frac{\phi^4}{216} - \ldots\right) \qquad (2.16)$$

$$Y = L_s \left(\frac{\phi}{3} - \frac{\phi^3}{42} + \frac{\phi^5}{1320} - \ldots\right) \qquad (2.17)$$

The angle subtended by the intermediate circular arc is equal to $(\Delta - 2\phi)$ in degrees.

The external distance E_s is computed as follows

$$E_s = (R+S) * (\sec\frac{\Delta}{2} - 1) + S \quad (2.21)$$

2.6.2 Unequal-Tangent Spiraled Circular Curve(Unsymmetrical)

In this case the left and right spirals connected to the circular curve are of unequal length. This will result in an unequal tangents lengths [5].

Referring to Fig. 2-8, if the lengths of left and right spirals are L_{s1} and L_{s2} respectively, then

$$\phi_1 = L_{s1}/2R$$

$$\phi_2 = L_{s2}/2R$$

Similarly

$$S_1 = Y_1 - R\,(1 - \cos\phi_1)$$

$$S_2 = Y_2 - R\,(1 - \cos\phi_2)$$

$$X_{o1} = X_1 - R\sin\phi_1$$

$$X_{o2} = X_2 - R\sin\phi_2$$

Where

$$X_1 = L_{s1}\,(1 - \frac{\phi_1^{\,2}}{10} + \frac{\phi_1^{\,4}}{216} \;\ldots\ldots)$$

$$Y_1 = L_{s1}(\frac{\phi_1}{3} - \frac{\phi_1^{\,3}}{42} + \frac{\phi_1^{\,5}}{1320} - \ldots\ldots)$$

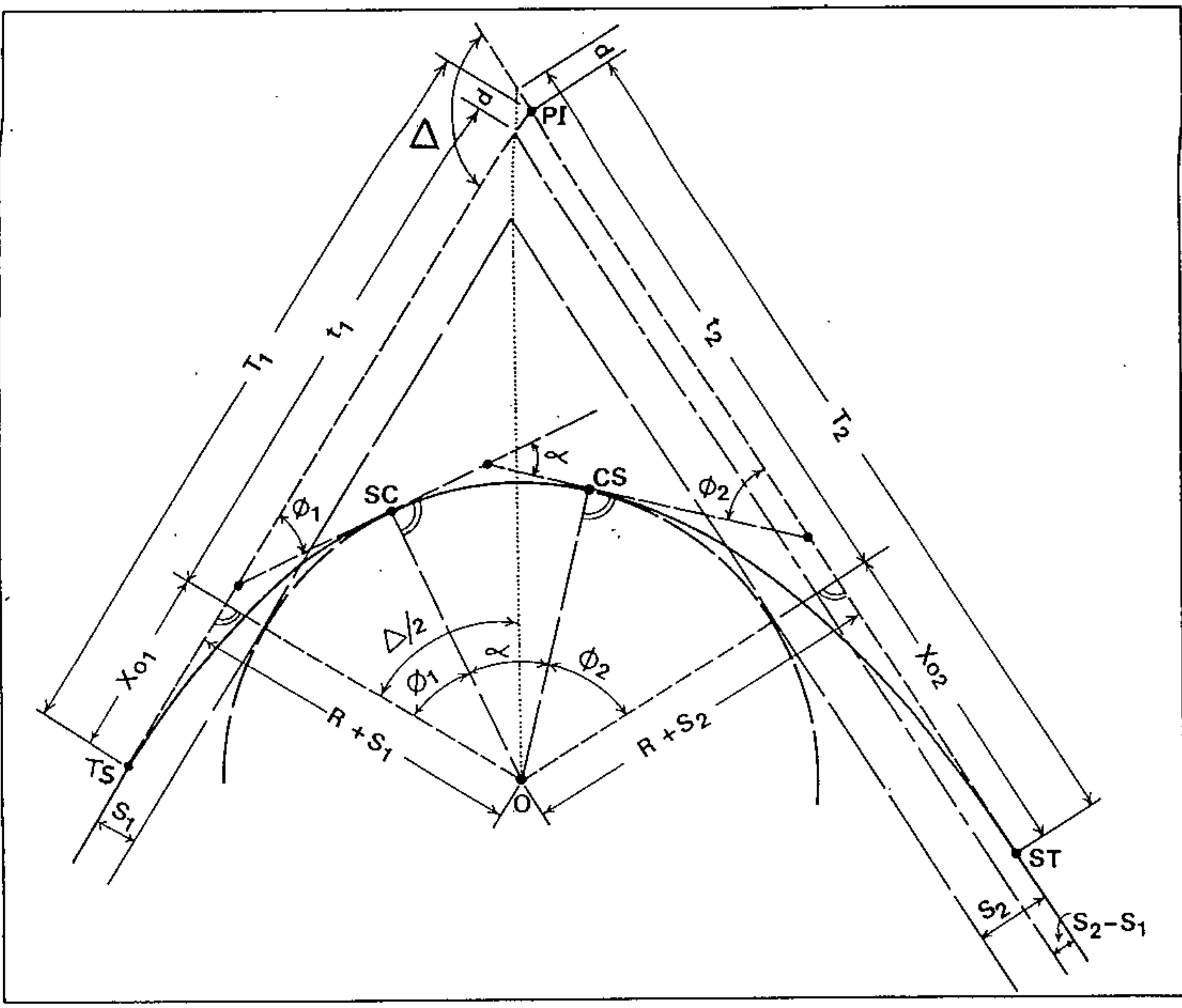


Figure (2-8): Unequal-Tangent Spiraled Sircular Curve [5]

$$X_2 = L_{s2} \left(1 - \frac{\phi_2^2}{10} + \frac{\phi_2^4}{216} \; ......\right)$$

$$Y_2 = L_{s2}\left(\frac{\phi_2}{3} - \frac{\phi_2^3}{42} + \frac{\phi_2^5}{1320} - .....\right)$$

In order to find the tangent distance, we note from Fig. 2-8 that the line joining the center O of the circular arc to the PI, will not bisect the total central angle Δ, thus

$$T_1 = X_{o1} + (R+S_1) \tan\frac{\Delta}{2} + d \qquad (2.22)$$

$$T_2 = X_{o2} + (R+S_2) \tan\frac{\Delta}{2} - d \qquad (2.23)$$

Where d is the displacement due to the unsymmetrical arrangements of the spirals and circular arc. The displacement d is found from the following equation [5]:

$$d = \frac{|S_2 - S_1|}{\sin \Delta} \qquad (2.24)$$

The central angle of the circular arc is equal to $(\Delta - \phi_1 - \phi_2)$.

2-7 DOUBLE SPIRAL CURVE

The double spiral curve joins two straight lines without the insertion of intermediate circular arc. It consists of two spirals having the same radius at their sharp ends. The two spirals can be of equal or unequal length.

2-7-1 Equal-Tangent Double Spiral Curve (Symmetrical)

In this case the two spirals are of equal length, thus the tangent distances are also equal [8,9].

referring to Fig. 2-9, we have

a- Spiral central angle , $\phi = \frac{\Delta}{2}$

b- $$X = L_s \left(1 - \frac{\phi^2}{10} + \frac{\phi^4}{216} - \ldots\ldots \right)$$

$$Y = L_s \left(\frac{\phi}{3} - \frac{\phi^3}{42} + \frac{\phi^5}{1320} - \ldots\ldots\ldots \right)$$

c- The tangent length is computed as follows

$$T = X + Y \tan \frac{\Delta}{2} \qquad (2.25)$$

d- The external distance is then computed,

$$E_s = \frac{Y}{\cos \Delta/2} \qquad (2.26)$$

2-7-2 Unequal-Tangent Double Spiral Curve (Unsymmetrical)

In this case, the left and right spirals are of unequal lengths which results in different tangents lengths. Referring to Fig. 2-10, if the length of left

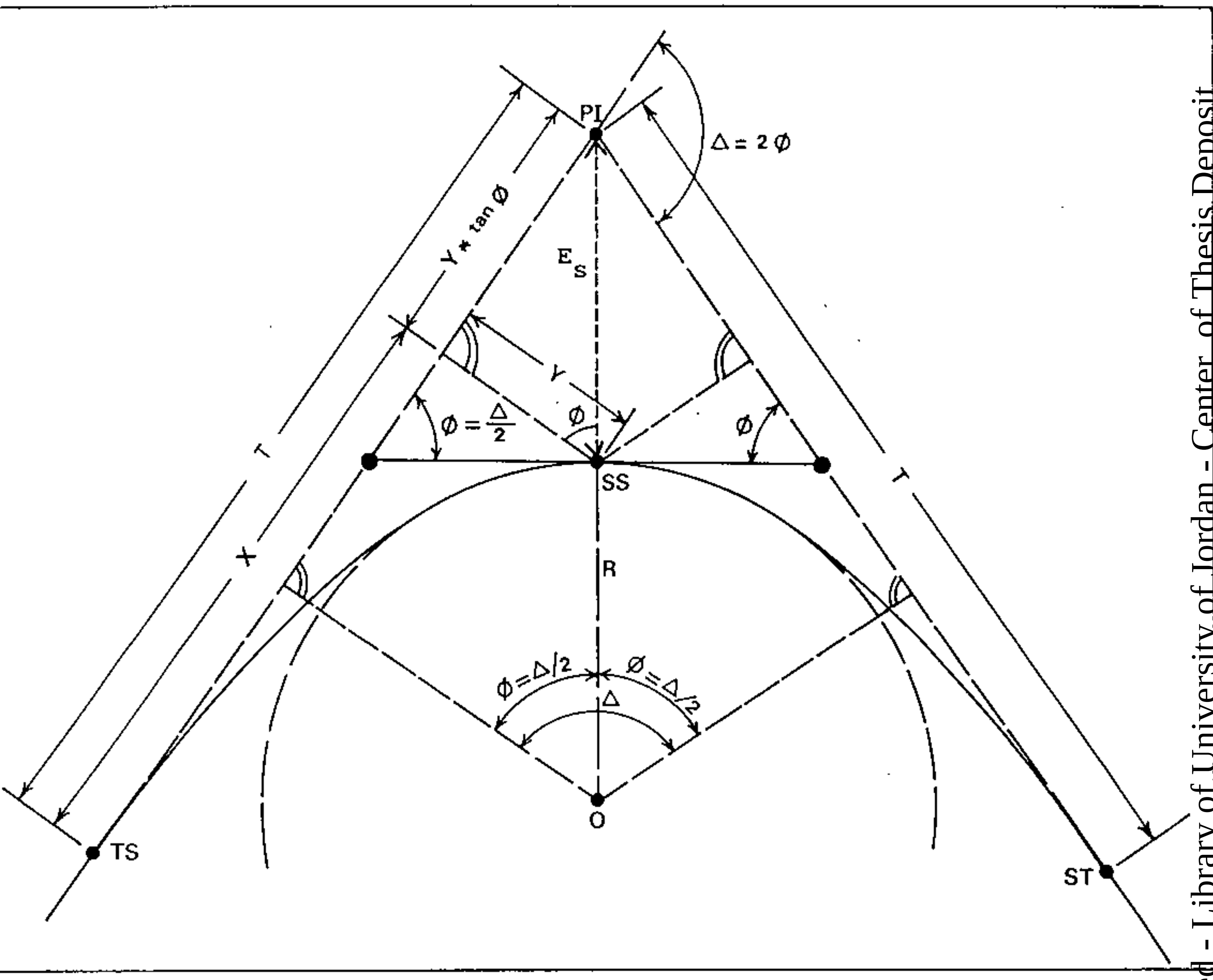


Figure (2-9): Equal-Tangent Double Spiral Curve

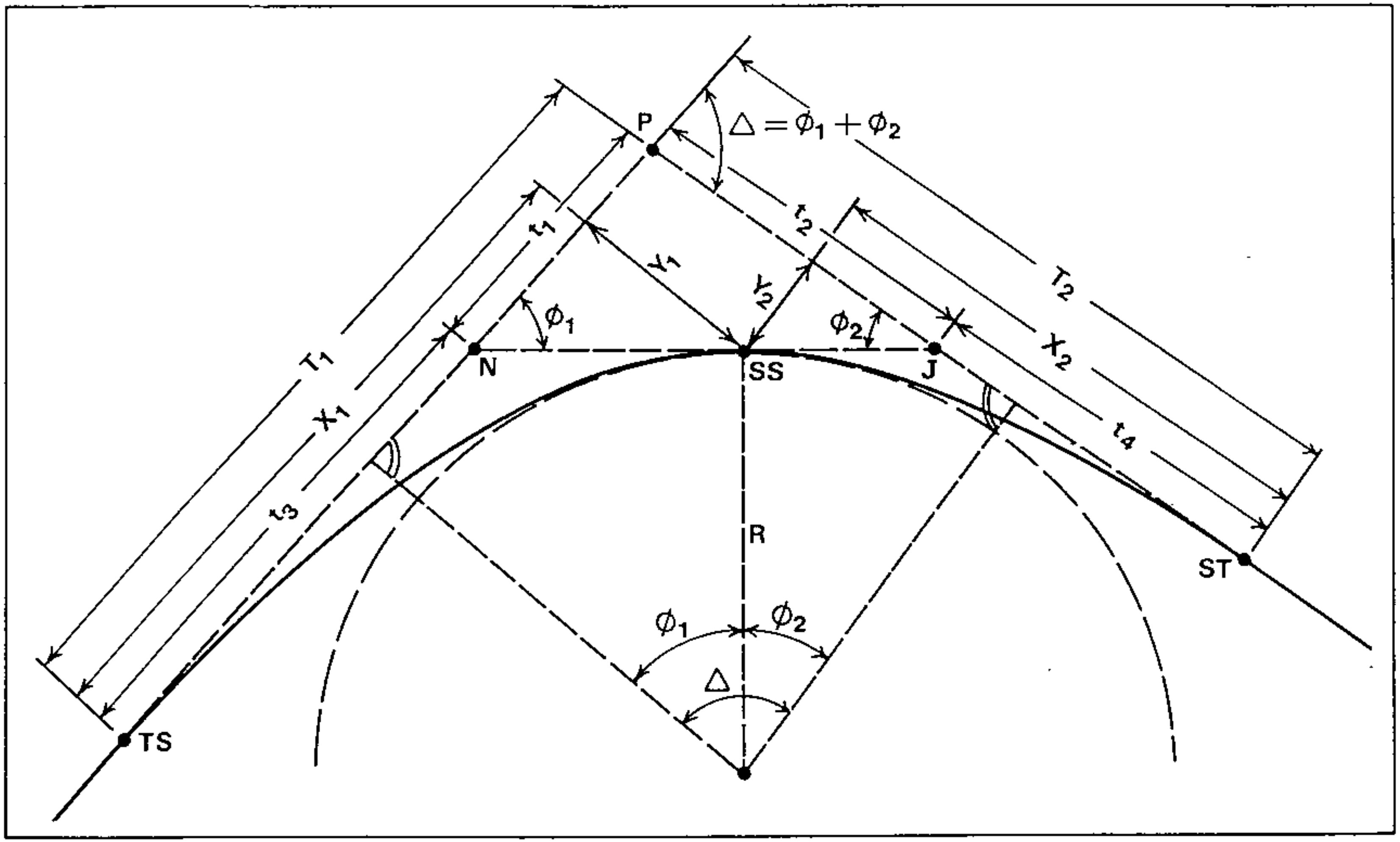


Figure (2-10): Unequal-Tangent Double Spiral Curve spiral L_{s1} is given and if the angle ϕ_1 between the back and common tangents is also known, then the following steps will be followed [4]:

a- $\phi_2 = \Delta - \phi_1$

b- Compute the radius at common tangency point SS,

$$\phi_1 = L_{s1}/2*R$$

or

$$R = L_{s1}/2*\phi_1$$

c- Compute the length of second spiral,

$$\phi_2 = L_{s2}/2*R$$

thus

$$L_{s2} = 2\ R\ \phi_2$$

d- Compute the parameters X_1, Y_1, X_2 and Y_2 for the two spirals using equations (2.16) and (2.17):

$$X_1 = L_{s1}\ (1 - \frac{\phi_1^2}{10} + \frac{\phi_1^4}{216}\ \ldots\ldots)$$

$$Y_1 = L_{s1}(\frac{\phi_1}{3} - \frac{\phi_1^3}{42} + \frac{\phi_1^5}{1320} - \ldots\ldots)$$

$$X_2 = L_{s2}\ (1 - \frac{\phi_2^2}{10} + \frac{\phi_2^4}{216}\ \ldots\ldots)$$

$$Y_2 = L_{s2}(\frac{\phi_2}{3} - \frac{\phi_2^3}{42} + \frac{\phi_2^5}{1320} - \ldots\ldots)$$

e- Find the length of common tangent as follows,

$$NJ = Y_1/\sin\phi_1 + Y_2/\sin\phi_2$$

By solving the triangle PNJ we get

$t_1 = PN = NJ \sin \phi_2/\sin \Delta$

$t_2 = PJ = NJ \sin \phi_1/\sin \Delta$

f- Compute the distances t_3 and t_4 shown in Fig. 2-10.

$t_3 = X_1 - Y_1/\tan \phi_1$

$t_4 = X_2 - Y_2/\tan \phi_2$

g- The tangents lengths can be determined as follows:

$$T_1 = t_1 + t_3 \qquad (2.27)$$

$$T_2 = t_2 + t_4 \qquad (2.28)$$

2-8 SPIRAL APPLIED TO EXISTING CIRCULAR CURVE

Suppose that we have an existing circular curve of radius R, and that the central part of this curve is to be maintained. The ends of the circular curve are to be replaced with spiral curves in order to improve the alinement of the curve [2].

To permit a spiral to be inserted between the original tangents and the circular arc, the existing circular curve must be compounded with a sharper curve, as shown in Fig. 2-11 and Fig. 2-12.

Given the spiral central angle desired ϕ , then we have

$$\text{Shift} = 0 = Y - R\ (1 - \cos \phi\ )$$

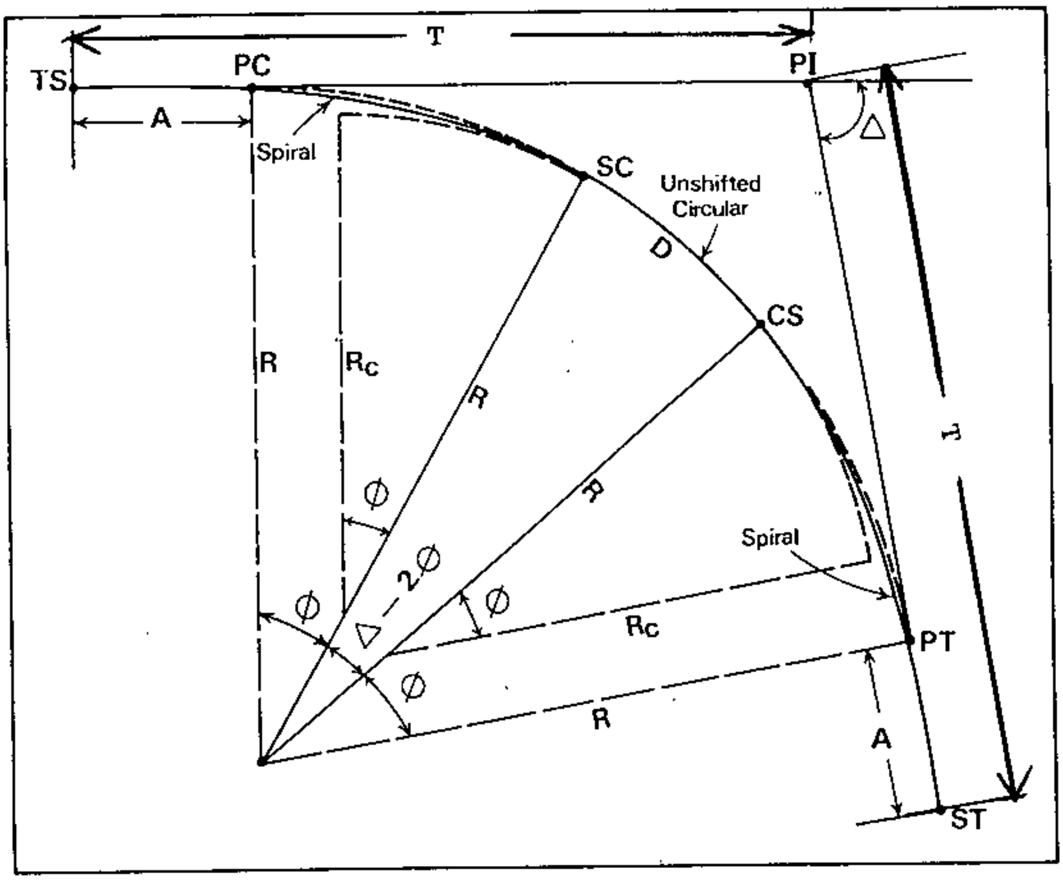


Figure (2-11): Spiral Applied To Existing Circular Curve [2]

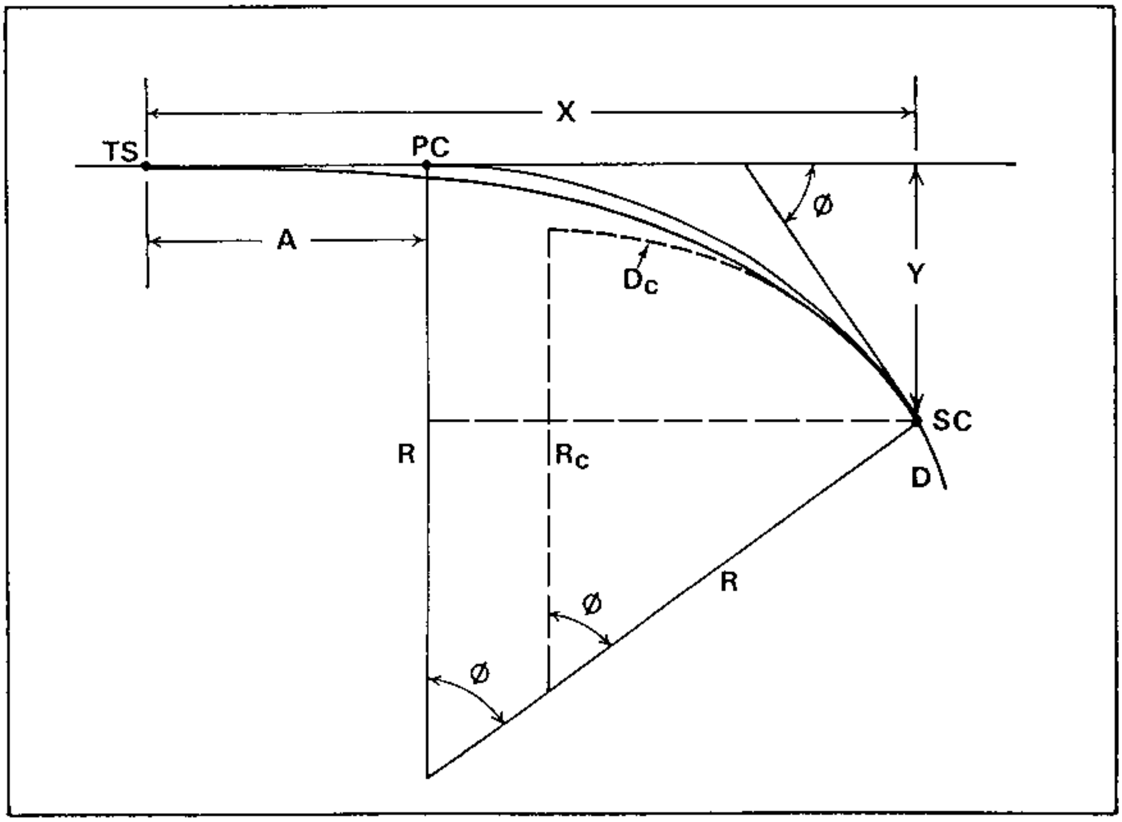


Figure (2-12): Showing Part of Figure (2-11) therefore

$$Y = R\ (\ 1 - \cos\phi\ )$$

If we have a spiral of unit length and a central angle of ϕ , then the local coordinates of its end points with respect to the TS are the following :

$$x_1 = 1\ (1 - \frac{\phi^2}{10} + \frac{\phi^4}{216} \ldots\ldots)$$

$$y_1 = 1\ (\frac{\phi}{3} - \frac{\phi^3}{42} + \frac{\phi^5}{1320} - \ldots.)$$

The length of the spiral needed is

$$L_s = \frac{Y}{y_1} = \frac{R(1-\cos\phi)}{y_1} \tag{2.29}$$

The radius of the sharpened circular arc is

$$R_c = L_s/2\phi$$

The distance A from the original PC to the TS point is found as follows

$$X = L_s\ x_1$$

$$A = X - R\sin\phi \tag{2.30}$$

Finally the new tangent length is

$$T = A + R\tan\Delta/2 \tag{2.31}$$

2-9 SPIRALED COMPOUND CURVES

When two arcs of different radii are joined together as in the case of compound curve, there will be an abrupt change in curvature. Such change in curvature may be easd by introducing a combining spiral. This combining spiral will provide a gradual change in curvature from one arc to the other.

Depending on the type of compound curve and on the number of required spirals, several cases may arise such as:

a- 2-Centered compound curve with one spiral.

b- 2-Centered compound curve with 3 spirals.

c- 3-Centered compound curve with 4 spirals.

d- 3-Centered compound curve with 5 spirals [2].

2-9-1 Two-Centered Compound Curve With One Connecting Spiral

Referring to Figures 2-13, 2-14 and 2-15 the following notations and symbols will be used [2]:

D_1, R_1 : Degree of curve and radius of left circular arc.

D_2, R_2 : Degree of curve and radius of right circular arc.

L_s : Length of combining spiral.

ϕ : Nominal spiral central angle.

S : The radial shift, that is the distance one branch is moved inward or outward at C (Figure 2-14)

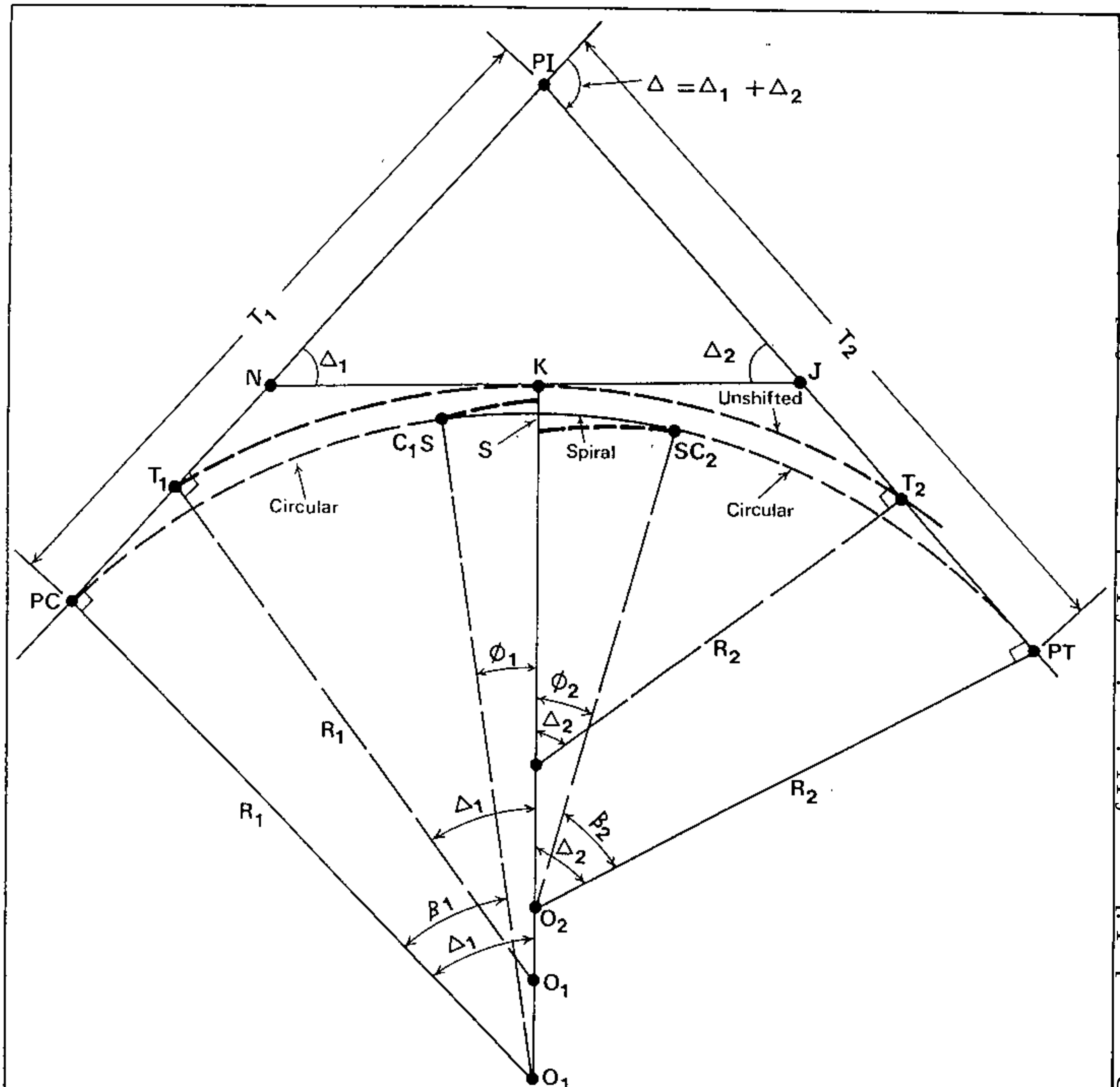


Figure (2-13): Two-Centered Compound Curve With One spiral Curve

l : The arc length of AP (Figure 2-13)

θ : Central angle of arc AP.

ϕ_1 : Central angle of left half of the spiral (Fig 2-14)

ϕ_2 : Central angle of right half of the spiral (Fig 2-14)

X,Y : Local coordinates of point SC_2 with respect to point C_1S as origin and line AV as X--axis (Figures 2-13 and 2-14)

δ : Deflection angle to any point on the spiral.

T_1,T_2: The total tangents lengths of the whole spiraled compound curve (Fig. 2-13).

T_3,T_4: The tangents lengths of the combining spiral AB_1 (Fig. 2-14)

Δ_1 : Central angle of the unshifted left circular arc (Fig 2-13)

Δ_2 : Central angle of the unshifted right circular arc.

Δ : Total deflection angle at PI (Fig 2-13)

β_1 : Central angle of shifted left circular arc,$= \Delta_1 - \phi_1$

β_2 : Central angle of shifted right circular arc,$= \Delta_2 - \phi_2$

The following steps will be considered in the design process of the spiraled compound curve [2]:

1- Compute the radial shift S (Fig. 2-13)

$$\phi_1 = L_s /2*R_1$$

$$\phi_2 = L_s/2*R_2$$

$$Y_1 = L_s \left(\frac{\phi_1}{3} - \frac{\phi_1^{\,3}}{42} + \frac{\phi_1^{\,5}}{1320} - \ldots\ldots \right)$$

$$Y_2 = L_s \left(\frac{\phi_2}{3} - \frac{\phi_2^3}{42} + \frac{\phi_2^5}{1320} - \ldots \right)$$

$$S_1 = Y_1 - R_1 (1 - \cos \phi_1)$$

$$S_2 = Y_2 - R_2 (1 - \cos \phi_2)$$

Therefore

Radial shift = $S = | S_1 - S_2 |$

2- Compute the tangents T_1 and T_2.

Considering the traverse PI,PC,O_1,O_2,PT,PI in Fig. 2-13 , we can write,

$\Sigma X = 0$

$$T_1 + T_2 \cos \Delta - R_2 \sin \Delta + (R_2 - R_1 - S) \sin \Delta_1 = 0$$

$\Sigma Y = 0$

$$T_2 \sin \Delta + R_2 \cos \Delta - (R_2 - R_1 - S) \cos \Delta_1 = 0$$

These two equations will be solved for T_1 and T_2 ,thus we get

$$T_2 = \frac{(R_2 - R_1 - S) \cos \Delta_1 - R_2 \cos \Delta}{\sin \Delta} \quad (2.33)$$

$$T_1 = R_2 \sin \Delta - T_2 \cos \Delta - (R_2 - R_1 - S) \sin \Delta_1 \quad (2.34)$$

3- For the combining spiral, derive a formula for the angle between the tangent at any point P and the tangent at point A (CS) :

a- Consider the case where we proceed from flatter to

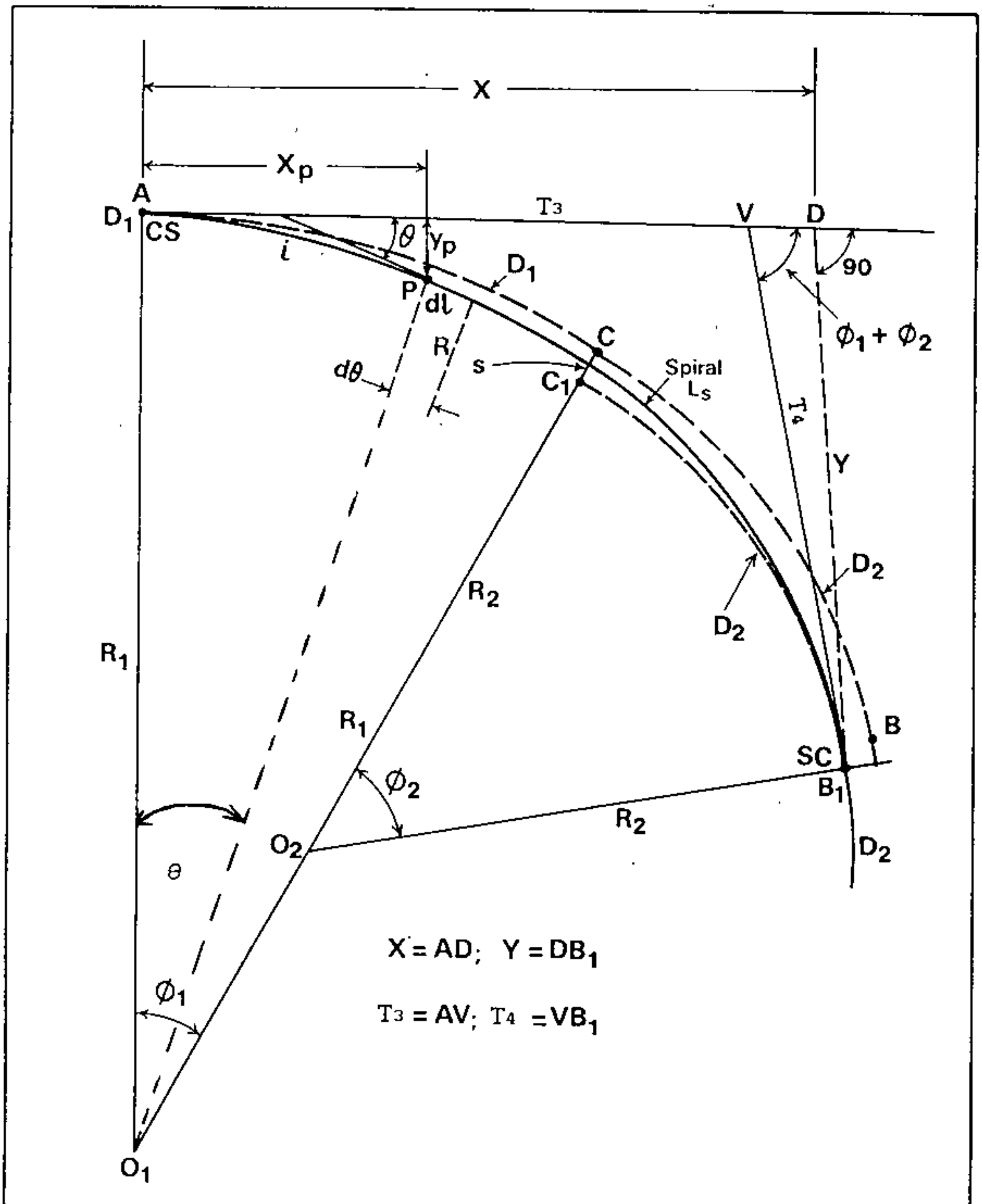


Figure (2-14): Combining Spiral (R1>R2) [2] sharper arc ($R_1 > R_2$ or $D_1 < D_2$) as shown in Fig. 2-13,

$$\text{Spiral central angle} = \phi = \phi_2 - \phi_1 = \frac{L_s}{2\,R_2} - \frac{L_s}{2\,R_1}$$

$$= \frac{L_s}{60}\,(D_2 - D_1)$$

Since the degree of curve D, is proportional to the distance from point A (CS), then we have

$$D = D_1 + k\,l \qquad (2.35)$$

At point B (SC), $D = D_2$ and $l = L_s$, thus we have

$$D_2 = D_1 + k\,L_s$$

$$k = \frac{D_2 - D_1}{L_s} \qquad (2.36)$$

By substituting the value of k in equation (2.35), we get

$$D = D_1 + \frac{l}{L_s} + (D_2 - D_1) \qquad (2.37)$$

Now, if we consider a sector at P , then we can write

$$d\theta = \frac{dl}{R} = \frac{D}{30}\,dl$$

or

$$D = 30 * \frac{d\theta}{dl} \quad \text{(in radian)}$$

Substituting the D value in equation (2.37) and rearranging, we get

$$d\theta = \frac{1}{30} (D_1 + (D_2 - D_1) * \frac{1}{L_s}) \, d_1$$

By integration we obtain

$$\theta = \frac{1}{30} * (D_1 + \frac{1}{2} * (D_2 - D_1) * \frac{1}{L_s}) \quad (2.38)$$

At point B (SC), $1=L_s$ and $\theta = \phi_1 + \phi_2 = \frac{L_s}{60} * (D_1 + D_2)$

but $\phi = \frac{L_s}{60} * (D_2 - D_1)$

thus

$$(D_2 - D_1) = 60 * \frac{\phi}{L_s}$$

Equation (2.38) can now be written as

$$\theta = \frac{D_1 \, 1}{30} + \phi * (\frac{1}{L_s})^2 \quad (2.39)$$

or

$$\theta = \frac{1}{R_1} + \phi * (\frac{1}{L_s})^2 \quad (2.40)$$

b- Consider the case where we proceed from the sharper to flatter arc ($R_1<R_2$ or $D_1>D_2$) as in Fig. 2-15. In this case the following equations are used :

$$D = D_1 - k \, 1$$

$$\theta = \frac{D_1 \, 1}{30} - \phi * (\frac{1}{L_s})^2$$

or

$$\theta = \frac{1}{R_1} - \phi * (\frac{1}{L_s})^2$$

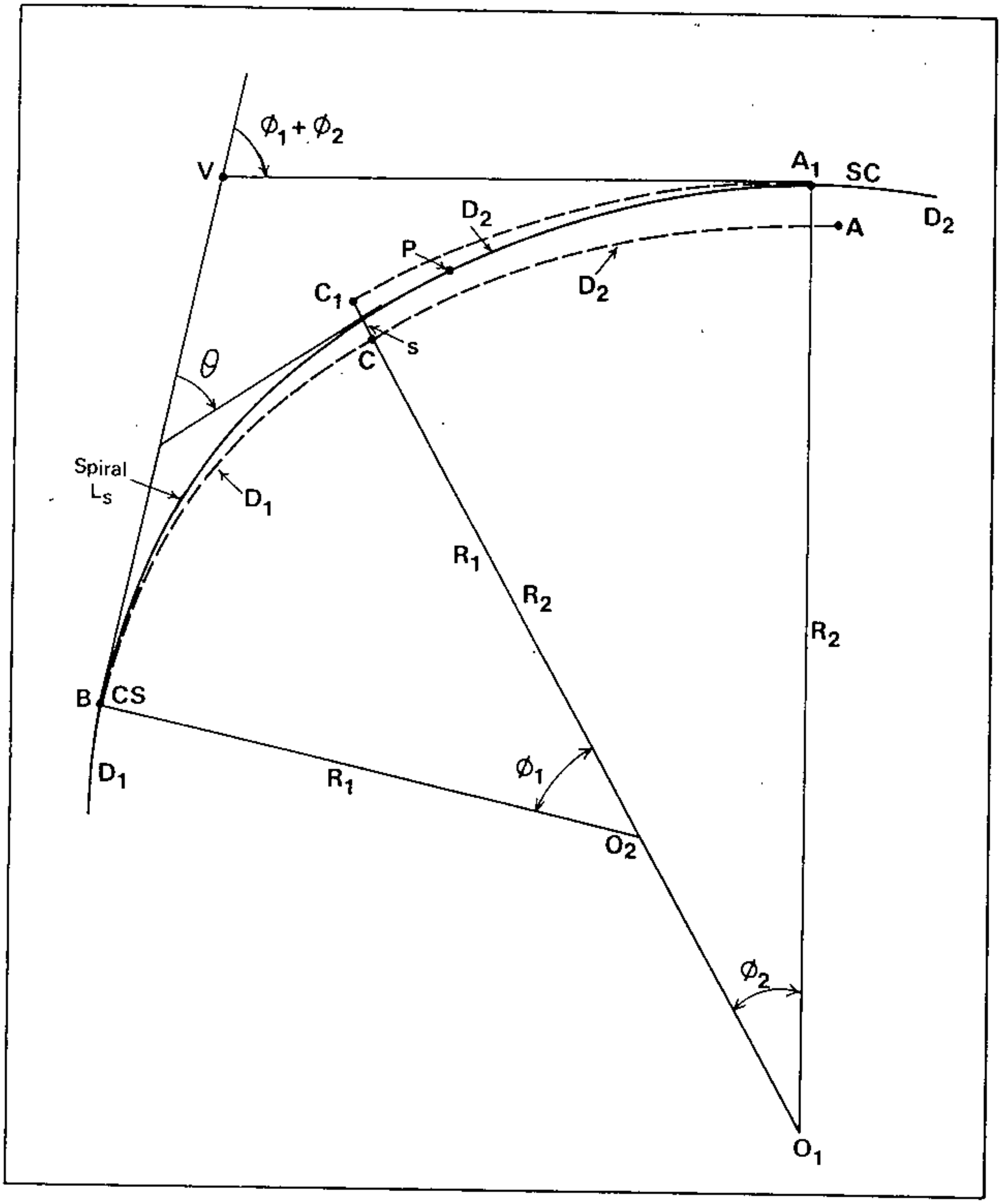


Figure (2-15): Combining Spiral (R1<R2)

4- Compute the local X and Y coordinates for any point P on the combining spiral with reference to point A (CS) as origin and tangent line AV as X-axis [2].

a- Consider the case where $R_1>R_2$ ($D_1<D_2$) as shown in Fig 2-14,

$$\theta = \frac{1}{30} * (D_1 + \frac{1}{2} * (D_2 - D_1) * \frac{1}{L_s})$$

assuming $j = [\frac{1}{2} * (D_2 - D_1) / D_1)] / L_s$

then

$$\theta = \frac{D_1 \; l}{30} * (1 + j*l)$$

or

$$\theta = \frac{D_1 \; l}{30} * (1 + k)$$

where

$K = j * l$

We have also from equation (2.39),

$$\theta = \frac{D_1 \; l}{30} + \phi * (\frac{l}{L_s})^2$$

For the differential element at P in Fig. 2-14

$dx = dl \sin \theta$

$dy = dl \cos \theta$

By expansion of sine and cosine functions and then substituting in the above differential equations , we get

$$dy = dl \left(\theta - \frac{\theta^3}{3\,!} + \frac{\theta^5}{5\,!} - \ldots\ldots\ldots \right)$$

$$dx = dl \left(1 - \frac{\theta^2}{2\,!} + \frac{\theta^4}{4\,!} - \ldots\ldots\ldots \right)$$

Finally, by substituting equation (2.40) in the above 2 equations then by integration and rearranging, we get the following two important equations :

$$x = l - \frac{1}{2\,!}\left(\frac{D_1}{30}\right)^2 l^3 \left[\frac{1}{3} + \frac{2}{4}k + \frac{1}{5}k^2\right] + \frac{1}{4\,!}\left(\frac{D_1}{30}\right)^4 l^5$$

$$* \left[\frac{1}{5} + \frac{4}{6}k + \frac{6}{7}k^2 + \frac{4}{8}k^3 + \frac{1}{9}k^4\right] + \ldots$$

$$+ \text{ negligible terms} \qquad (2.41)$$

$$y = \left(\frac{D_1}{30}\right) l^2 \left[\frac{1}{2} + \frac{1}{3}k\right] - \frac{1}{3\,!}\left(\frac{D_1}{30}\right)^3 l^4 \left[\frac{1}{4} + \frac{3}{5}k\right.$$

$$\left. + \frac{3}{6}k^2 + \frac{1}{7}k^3\right] + \frac{1}{5\,!}\left(\frac{D_1}{30}\right)^5 l^6 \left[\frac{1}{6} + \frac{5}{7}k + \frac{10}{8}k^2\right.$$

$$\left. + \frac{10}{9}k^3 + \frac{5}{10}k^4 \; \frac{1}{11}k^5\right] + \ldots \text{ negligible terms} \qquad (2.42)$$

These two equations give the local coordinates x & y of any point P on the combining spiral with respect to point CS as an origin and the tangent through it as an X-axis [2].

b- Consider the case where $R_1<R_2$ $(D_1>D_2)$ as shown in Fig 2-15,

$$\theta = \frac{D_1*l}{30}\,(1 - j*l)$$

$$= \frac{D_1*l}{30}\,(1 - k)$$

where

$$j = [\tfrac{1}{2} * (D_2 - D_1) / D_1)] / L_s$$

$$k = - j*l$$

Thus equations (2.41) and (2.42) for x & y can be used but with negative value of k.

2-9-2 Two-Centered Compound Curve With Three Spirals

This type of horizontal curves consists of a succession of spiral-circular-spiral-circular-spiral curves. In Fig.2-16 a non-spiraled compound curve is shown, and the same curve provided with three spirals is shown as well.

If the length of entry spiral is denoted by L_{s1}, and that of the exit spiral by L_{s3}, then the length of connecting spiral is the difference between the exit and entry spirals lengths, or in symbols [9]:

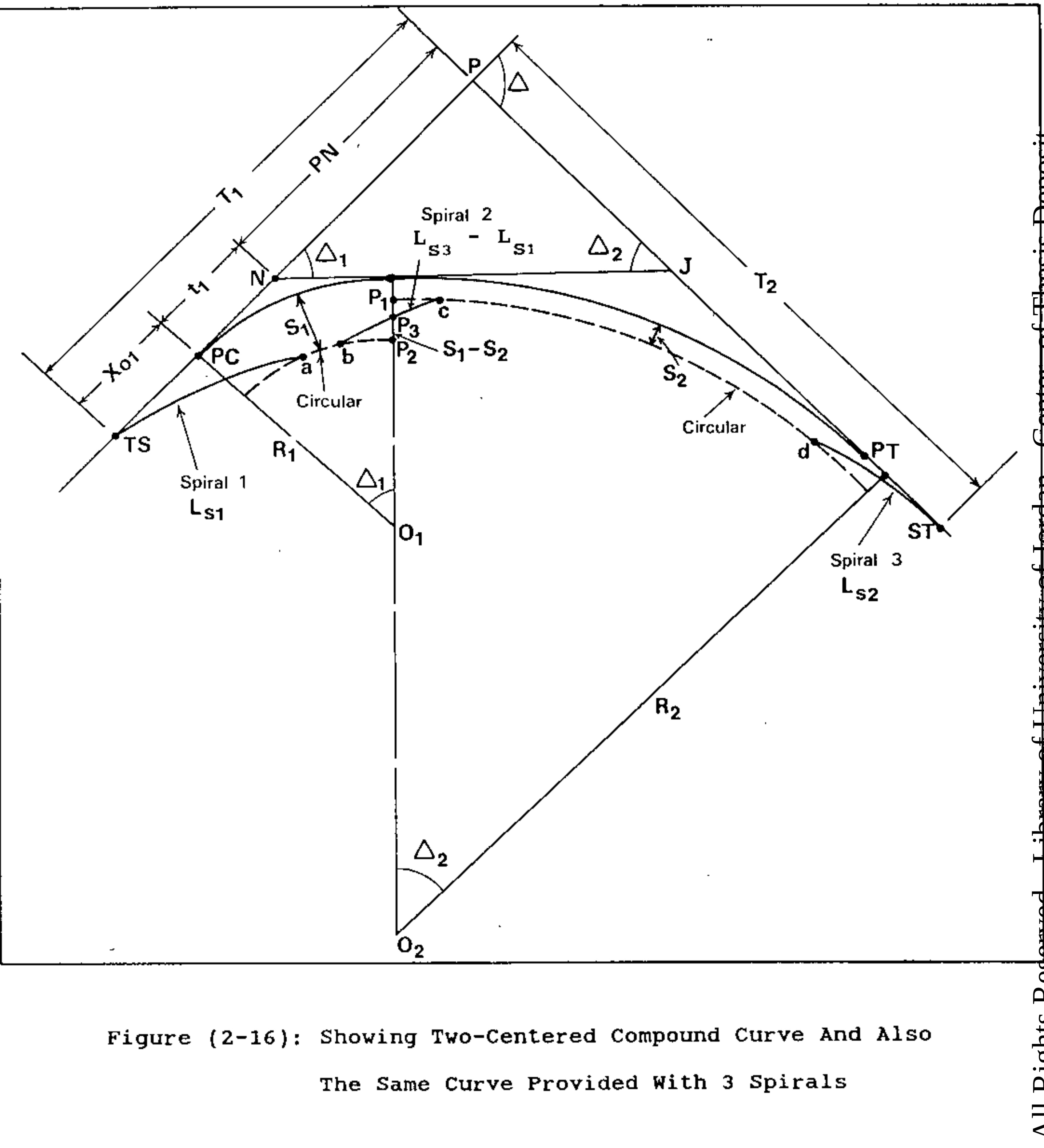


Figure (2-16): Showing Two-Centered Compound Curve And Also The Same Curve Provided With 3 Spirals

$$L_{s2} = L_{s3} - L_{s1}$$

The following steps will be considered in the design process of this type of curve :

1- By examining Fig. 2-17 we can compute the values of the shown angles as follows:

$$\Delta = \Delta_1 + \Delta_2$$

$$\phi_1 = L_{s1}/2\ R_1$$

$$\phi_2 = L_{s3}/2\ R_2$$

$$\phi_3 = L_{s2}/2\ R_1$$

$$\phi_4 = L_{s2}/2\ R_2$$

also

$$\Delta_1 = \phi_1 + \phi_3 + \beta_1$$

$$\Delta_2 = \phi_2 + \phi_4 + \beta_2$$

Where

β_1 and β_2 are the central angles for the left and right circular arcs.

2- The radial shift that must be applied to the circular arcs, to permit the insertion of the spirals is now computed.

$$S_1 = Y_1 - R_1\ (\ 1 - \cos\ \phi_1\ )$$

$$S_2 = Y_2 - R_2\ (\ 1 - \cos\ \phi_2\ )$$

where

S_1 & S_2 are the shifts of the entry and exit spirals.

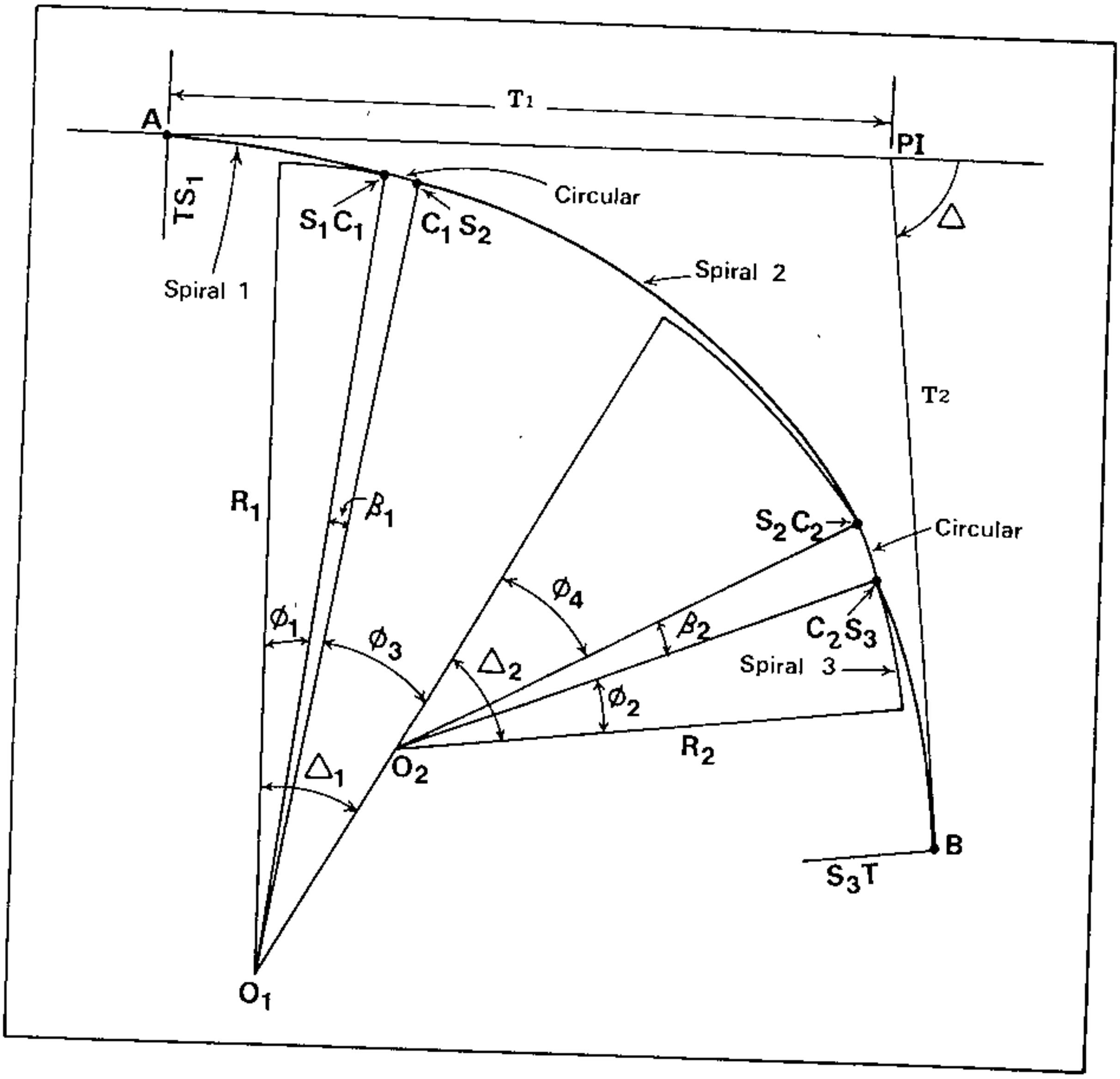


Figure (2-17): Two-Centered Compound Curve With 3 Spirals

$$Y_1 = L_{s1} \left(\frac{\phi_1}{3} - \frac{\phi_1^3}{42} + \frac{\phi_1^5}{1320} - \ldots\ldots \right)$$

$$Y_2 = L_{s3} \left(\frac{\phi_2}{3} - \frac{\phi_2^3}{42} + \frac{\phi_2^5}{1320} - \ldots\ldots \right)$$

3- The tangents lengths are computed in the following way:

a- The length of common tangent NJ is found first (Fig. 2-16),

$$NJ = (R_1 + S_1) \tan \frac{\Delta_1}{2} + (R_2 + S_2) \tan \frac{\Delta_2}{2}$$

b- By solving the triangle PNJ ,compute length of PN,

$$PN = NJ * \frac{\sin \Delta_2}{\sin \Delta}$$

$$PJ = NJ * \frac{\sin \Delta_1}{\sin \Delta}$$

c- The tangent length of the unshifted circular arc is then computed,

$$t_1 = N\text{-}PC = (R_1 + S_1) \tan \frac{\Delta_1}{2}$$

d- Compute the distance X_{o1} between the unshifted PC of the left circular arc and the point TS_1 (beginning point of curve),

$$X_1 = L_{s1} \left(1 - \frac{\phi_1^2}{10} + \frac{\phi_1^4}{216} - \ldots\ldots \right)$$

$$X_{o1} = PC\text{-}TS_1 = X_1 - R_1 \sin \phi_1$$

e- The overall tangent length is thus, equal to the sum of PN, t_1 and X_{o1} ,

$$T_1 = PN + t_1 + X_{o1}$$

$$T_1 = [(R_1 + S_1)\ \tan\frac{\Delta_1}{2} + (R_2 + S_2)\ \tan\frac{\Delta_2}{2}]*\frac{\sin\Delta_2}{\sin\Delta}$$

$$+ (R_1 + S_1)*\ \tan\frac{\Delta_1}{2} + X_1 - R_1*\sin\phi_1 \qquad (2.44)$$

The other tangent T_2, is computed in a similar manner.

4- Find the length of circular arcs,

$$L_1 = \pi * R_1 * (\Delta_1 - \phi_1 - \phi_3)/180$$

$$L_2 = \pi * R_2 * (\Delta_2 - \phi_2 - \phi_4)/180$$

5- For the combining spiral that connects the two circular arcs, the local coordinates of point S_2C_2 with respect to point C_1S_2 as an origin and the tangent at C_1S_2 as an X-axis can be found as follows:

From Fig. 2-16, the radial shift S is found first,

$$S = S_2 - S_1$$

Then from Fig. 2-18, solve Traverse $C_1S, V, S_2C_2, O_2, O_1, C_1S_2$ for x and y :

$$\Sigma X = 0$$

$$(R_2 - R_1 - S)\ \sin\phi_3 + R_2 \sin(\phi_3 + \phi_4) - x = 0$$

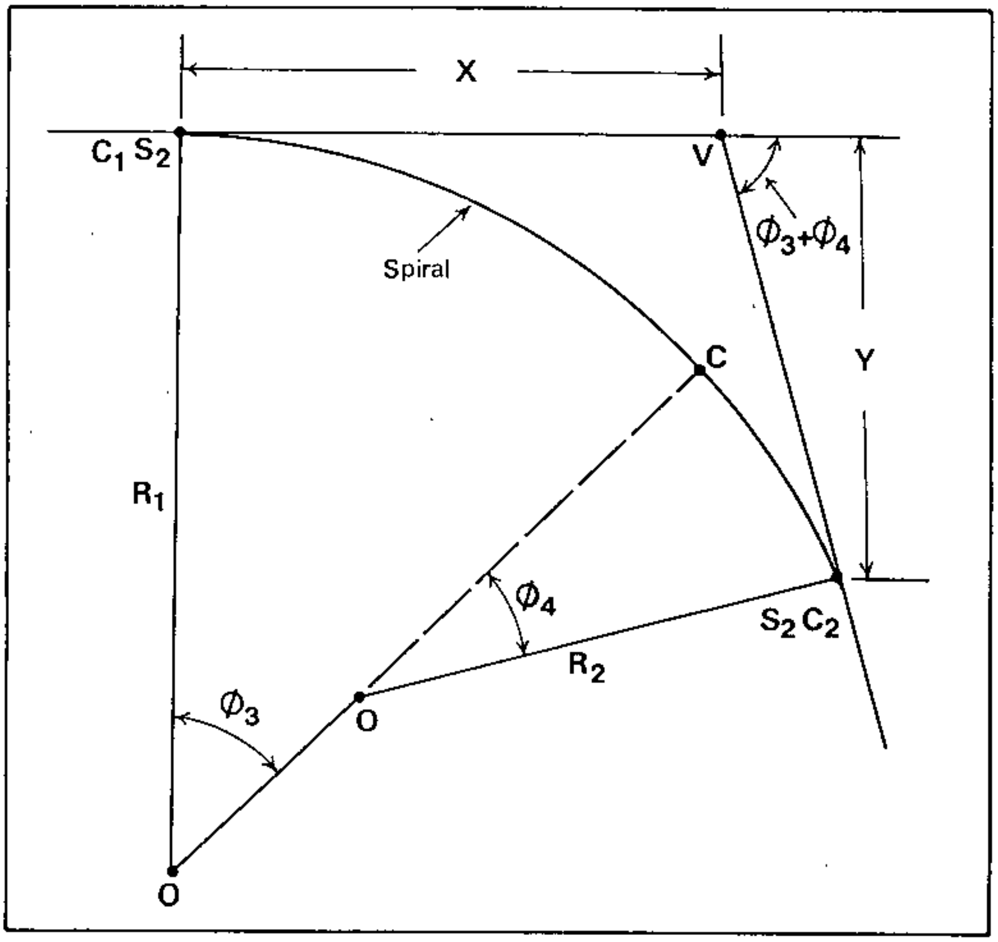


Figure (2-18): Showing how to compute coordinates of the end point of the combining spiral of the Two-centred compound curve with 3 spirals.

$$x = (R_1 - R_2 - S)\ \sin \phi_3 + R_2 \sin (\phi_3 + \phi_4) \qquad (2.45)$$

$$\Sigma\ Y = 0$$

$$R_1 - (R_1 - R_2 - S)\ \cos \phi_3 - R_2 \cos (\phi_3 + \phi_4) - y = 0$$

$$y = R_1 - (R_1 - R_2 - S)\ \cos \phi_3 - R_2 \cos (\phi_3 + \phi_4) \qquad (2.46)$$

The values of x and y represent the local coordinates of the end point of the intermediate spiral with respect to its beginning point [2,9].

2-10 SPIRALED REVERSE CURVES

This type of curve consists of a succession of spiral - circular - reverse spiral - circular - spiral curves. the S-curve that connects the two opposing circular arcs is made of two spirals having at their common point of origin the radius R equals infinity and a common tangent (Fig. 2-19 and 2-20).

The two spirals of the S-curve generally have similar parameters so that a steadily reversing route is obtained. Refering to fig (2-19), and assuming that L_{s1}, L_{s2}, L_{s3}, and L_{s4} represent the length of spirals. Assuming also that the distance between PI_1 and PI_2 is fixed and equal to AB. Then any of the following 2 cases will be resulted[5]:

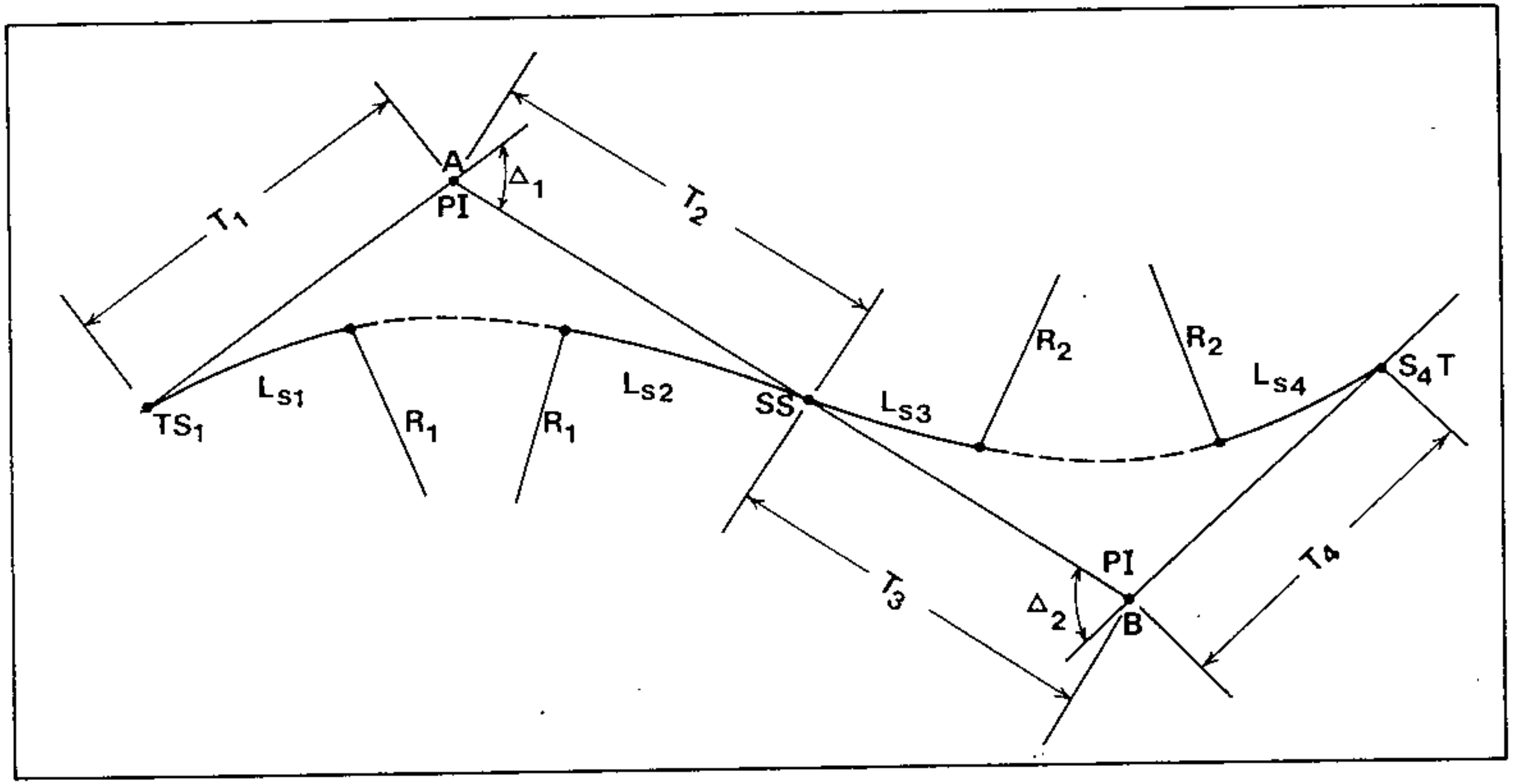


Figure (2-19): Spiraled Reverse Curve

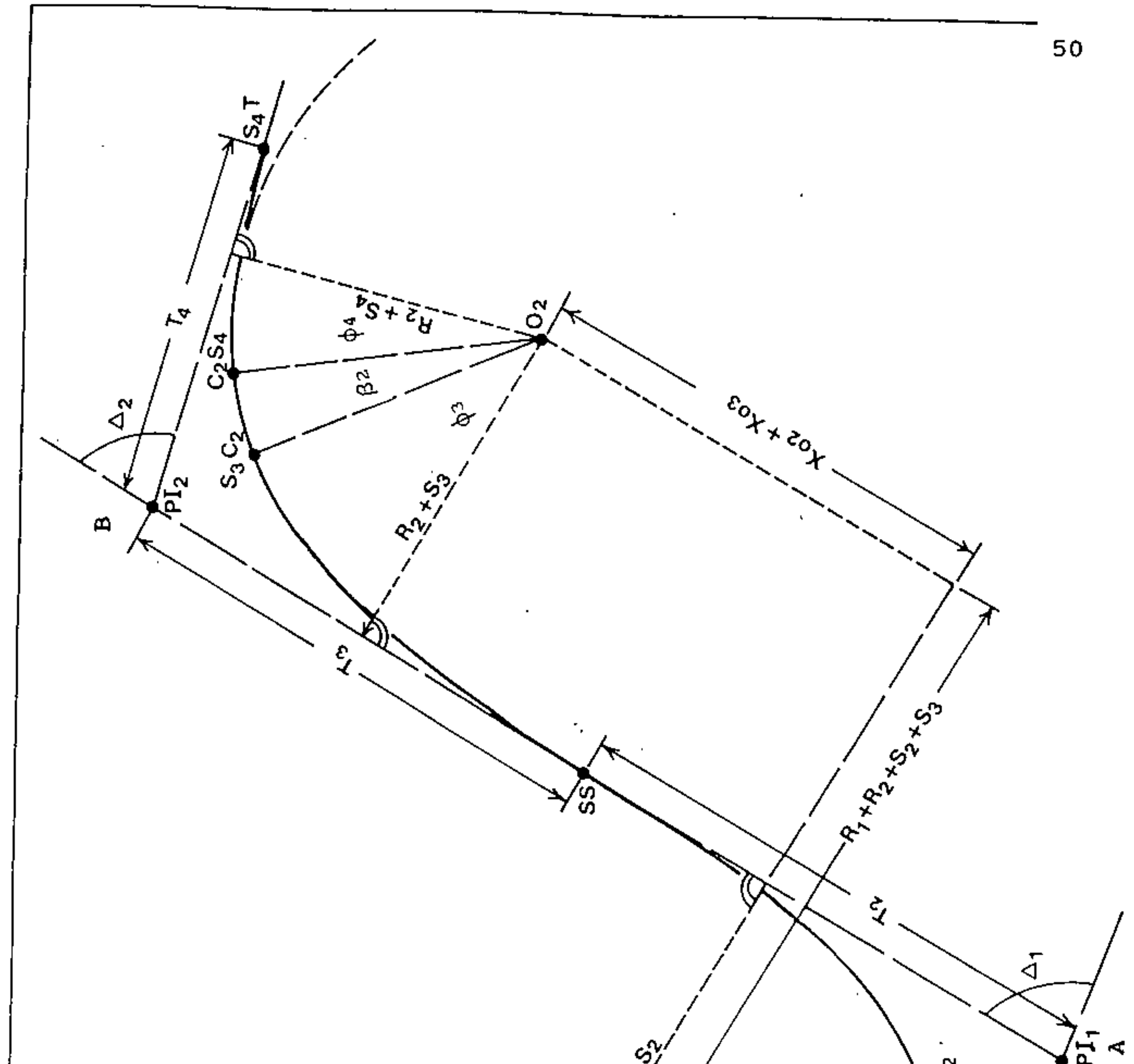


Figure (2-20): Spiraled Reverse Curve (Detailed) [5]

Case 1 : L_{s3} not equal L_{s4} (Fig. 2-20)

It will be assumed that $L_{s1}, L_{s2}, L_{s3}, R_1, R_2$ are given and it is required to find L_{s4} . We compute the parameters of each spiral ($\phi_1, \phi_2, \phi_3, X_1, X_2, X_3, Y_1, Y_2, Y_3, X_{o1}$, X_{o2}, X_{o3}, S_1, S_2, S_3) using the following equations:

$$X = L_s \left(1 - \frac{\phi^2}{10} + \frac{\phi^4}{216} - \ldots\ldots. \right)$$

$$Y = L_s \left(\frac{\phi}{3} - \frac{\phi^3}{42} + \frac{\phi^5}{1320} - \ldots.. \right)$$

$$X_o = X - R \sin \phi$$

$$S = Y - R (1 - \cos \phi)$$

The next step is to compute tangent distances using equations (2.22) & (2.23),

$$T_1 = X_{o1} + (R_1 + S_1) \tan \Delta_1/2 + \frac{S_2 - S_1}{\sin \Delta_1}$$

$$T_2 = X_{o2} + (R_1 + S_2) \tan \Delta_1/2 - \frac{S_2 - S_1}{\sin \Delta_1}$$

Since the distance between PI_1 and PI_2 is known, then

$$T_3 = AB - T_2$$

The length of the fourth spiral will be determined using the governing tangent length T_3. But before that the shift S_4 must be found. We have

$$T_3 = X_{o3} + (R_2 + S_3)\ \tan \Delta_2/2 + \frac{S_4 - S_3}{\sin \Delta_2}$$

assuming $H = \frac{S_4 - S_3}{\sin \Delta_2}$

then

$$H = T_3 - X_{o3} - (R_2 + S_3)\ \tan \Delta_2/2$$

Note that all variables are known except H (see the section of unequal-tangent spiral circular curve (2-6-2)).

$$S_4 = H \sin \Delta_2 + S_3$$

Now from S_4, the length of spiral L_{s4} will be found by the following method:

$$Y = L_s \left(\frac{\phi}{3} - \frac{\phi^3}{42} + \frac{\phi^5}{1320} - \ldots \right)$$

$$\phi = L_s/2R$$

therefore

$$Y = L_s \left(\frac{L_s}{6\ R} - \frac{L_s^3}{336\ R^3} + \ldots\ldots\ldots \right) \qquad (2\text{-}47)$$

The amount of shift is computed by the formula below:

$$S = Y - R\ (1 - \cos \phi) \qquad (2.19)$$

The cosine function is then expanded,

$$\cos \phi = 1 - \frac{\phi}{2\ !} + \frac{\phi}{4\ !} - \ldots\ldots\ldots$$

since $\phi = L_s /2*R$, then we have

$$\cos \phi = 1 - \frac{L_s^{\,2}}{8\ R} + \frac{L_s^{\,4}}{192\ R^3} + \ldots\ldots\ldots$$

Now by substituting the values of Y from equation (2.47) and cos ϕ from the above equation into equation (2.19), we get

$$S = L_s\left(\frac{L_s}{6R} - \frac{L_s}{336\ R^3} + \ldots\right) - R\left(1 - 1 + \frac{L_s}{8\ R} - \frac{L_s}{192\ R^4}\right.$$

$$+\ldots\ldots \text{ negligible terms}$$

By rearranging and omitting the negligible terms we get

$$S = \frac{L_s^2}{24\ R} - \frac{L_s^4}{2688\ R^3} \qquad (2.48)$$

In this case, $S = S_4$, $L_s = L_{s4}$, $R=R_2$.

Therefore L_{s4} can be found by solving the above equation by trial and error [4,1].

<u>Case</u> <u>2</u> : where the spiral length $L_{s3} = L_{s4}$

In this case L_{s3} will be assumed equal to L_{s4} , thus $S_4 = S_3$.

$$T_3 = AB - T_2$$

$$X = L_s\left(1 - \frac{\phi^2}{10} + \frac{\phi^4}{216}\ \ldots\ldots\right)$$

but $\phi = L_s/2R$

therefore

$$X = L_s \left(1 - \frac{L_s^2}{40\ R^2} + \text{....negligible terms..} \right)$$

Also $X_o = X - R \sin \phi$

Expanding the sine function as below we get :

$$\sin \phi = \phi - \frac{\phi^3}{3\,!} + \frac{\phi^5}{5\,!} - \text{.............}$$

$$= \frac{L_s}{2\ R} - \frac{L_s^3}{48\ R^3} + \text{......negligible terms}$$

Substituting the values of sin ϕ and X in the equation of X_o to get:

$$X_o = L_s \left(1 - \frac{L_s^2}{40\ R^2} + ..\right) - R \left(\frac{L_s}{2\ R} - \frac{L_s^3}{48\ R^3} + \text{....} \right)$$

$$= \frac{L_s}{2} - \frac{L_s^3}{240\ R^2} + \text{..negligible terms} \qquad (2.49)$$

However

$$T = (R + S) \tan \Delta/2 + X_o$$

Substituting the S value from equation (2.48) and the X_o value from eq (2.49), in the above equation we get the following equation:

$$T = \left(R + \frac{L_s^2}{24\,R} - \frac{L_s^4}{2688\,R^2}\right) \tan \Delta/2 + \frac{L_s}{2} - \frac{L_s^3}{240\,R^2} \qquad (2.50)$$

In this case $T' = T_3 = T_4 = AB - T_2$ and $L_s = L_{s3} = L_{s4}$, Therefore the equation will be solved for L_s by trial and error.

CHAPTER 3

MATHEMATICAL MODEL

3-1 Introduction

In this chapter, the following mathematical models for the design of horizontal curves will be discussed, in addition to traverse adjustment :

1- Simple Circular Curves.

2- Spiraled Circular Curves.

a- Equal-Tangent Spiraled Circular curves

b- Unequal-Tangent Spiraled Circular Curves

3- Spiral applied to an Existing Circular Curve

4- Double Spiral with no intermediate circular arc.

a- Equal-Tangent double spiral curve.

b- Unequal-Tangent double spiral curve.

5- Compound Curves.

a- Two-Centered compound curves.

b- Three-Centered compound curves.

6- Spiraled Compound Curves.

a- Two-Centered compound curve with 1 connecting Spiral

b- Two-Centered compound curve with 3 connecting Spirals

7- Reverse Curves.

a- Simple Reverse Curves.

b- Spiraled Reverse Curves.

3-2 TRAVERSE ADJUSTMENT

In order to design the above mentioned types of horizontal curves by fixing coordinates, it is necessary to know the coordinates of the point of intersection PI, and the azimuths of the back and forward tangents of the curve.

If a series of points of intersections that constitute a suggested route of a certain highway are given, then the absolute coordinates of these PIs should be established using the ordinary traverse adjustment. For this case, we have a closed connecting traverse that must be connected at the start and end points with ground control points of known coordinates or a combination of control points and sides of known azimuth [1].

The process of computing the absolute coordinates for a series of PIs that make the traverse, proceeds as follows (see Fig. 3-1) :

1- Determine what type of ground control is available.

Two types of ground control are usually possible:

a- Two control points of known coordinates are given at

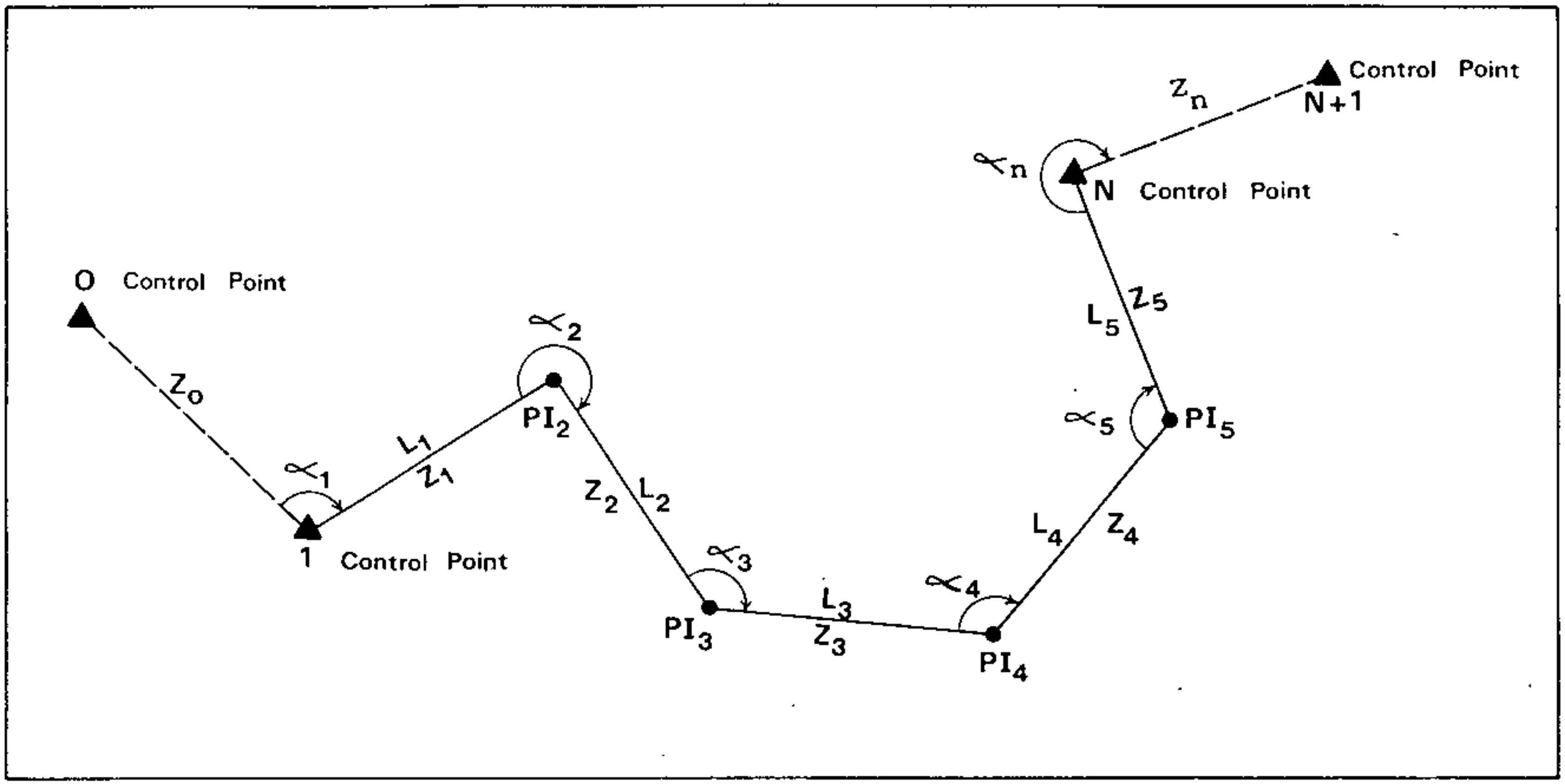


Figure (3-1): Connecting Traverse

each of the start and end of the connecting traverse. For this case the control azimuths Z_o and Z_n are computed at the starting and ending sides of the traverse. The azimuth is computed as follows :

$$Z = \tan^{-1} \frac{\Delta X}{\Delta Y} \qquad (3.1)$$

b- One control point of known coordinates and one control side of known azimuth are given at each of the start and end stations of the traverse .

2- Measure the length of all courses ($L_1, L_2, L_3, \ldots L_{n-1}$) and the clockwise horizontal angles ($\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_n$) . The horizontal angle is measured from the preceding course to the next one.

3- Compute the unadjusted azimuth of all courses . The azimuth of any course can be computed using the formula shown below (see Fig. 3-2):

$$Z_i = Z_{i-1} + \alpha_i \pm 180 \qquad (3.2)$$

where

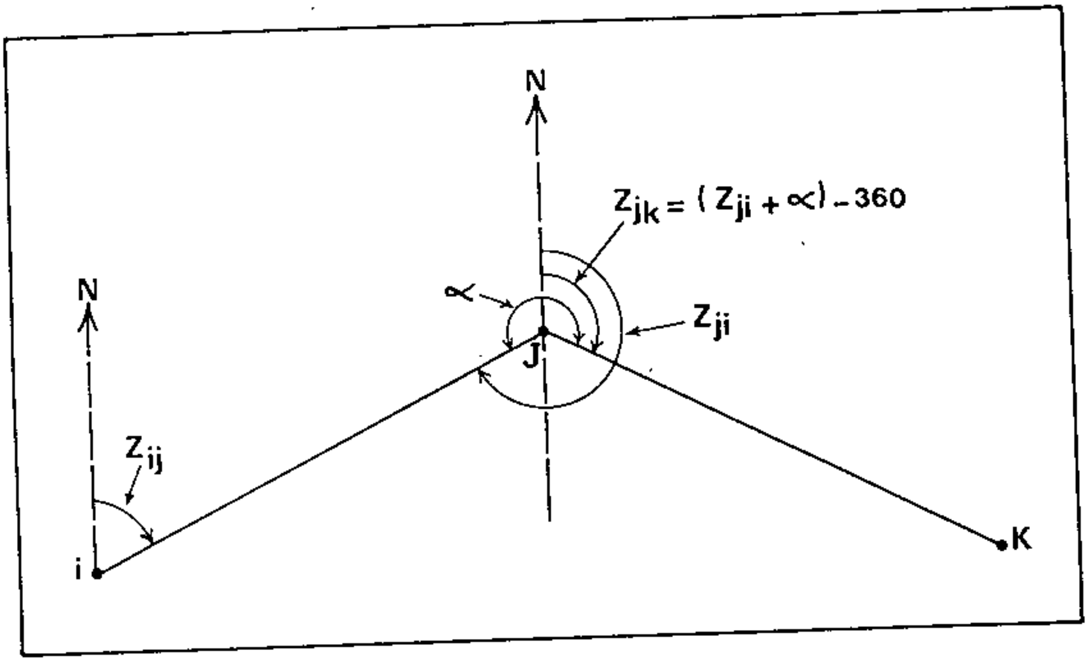
Z_i : azimuth of course i.

Z_{i-1} : azimuth of preceding course.

α_i : clockwise angle measured from preceding course (i-1) to the next course i.

If $Z_i > 360$ then

$$(Z_i)_{final} = Z_i - 360$$



Azimuth of side ij = Z_{ij}

Azimuth of side ji = Z_{ij} + 180

Azimuth of side jk = Z_{jk} = Z_{ij} + 180 + α

or

Azimuth of side jk = Z_{jk} = Z_{ij} − 180 + α

If Z_{jk} > 360, then

$(Z_{jk})_{final} = Z_{jk} - 360$

Figure (3-2): Azimuth Computations Of Traverse Sides

4- Determine the closure error in azimuth. The closure error is the difference between the computed and known azimuth of last control side.

$$\text{closure error} = \varepsilon = (Z_n)_{computed} - (Z_n)_{known} \qquad (3.3)$$

where n is the number of measured angles.

5- Compute the adjusted azimuth of all courses. The adjusted azimuth of any course is found by the following formula:

$$(Z_i)_{adjusted} = (Z_i)_{unadjusted} - \frac{i}{n} * \varepsilon \qquad (3.4)$$

Where

i : The number (order) of measured angle.

n : Number of all measured angles.

6- Compute preliminary coordinates of the points of intersections PIs . For any PI ,the preliminary coordinates are computed as follows (see Fig. 3-3) :

$$X_j = X_i + L_{ij} * \sin(Z_{ij}) \qquad (3.5)$$

$$Y_j = Y_i + L_{ij} * \cos(Z_{ij}) \qquad (3.6)$$

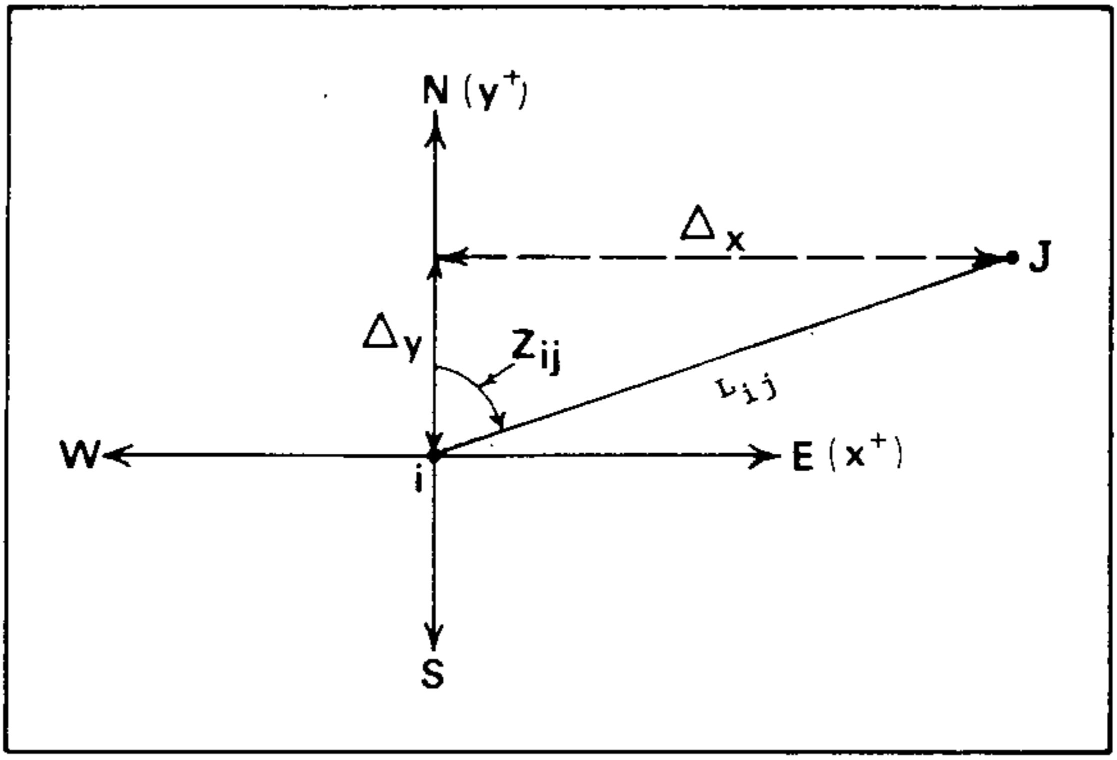
Where

(X_j, Y_j) : Required coordinates of point j .

(X_i, Y_i) : Coordinates of preceding point i .

L_{ij} : Length of course ij

Z_{ij} : Azimuth of course ij .



Length of side ij = L_{ij} = $(\Delta X^2 + \Delta Y^2)^{1/2}$

Azimuth of side ij = Z_{ij} = $\tan^{-1} \frac{\Delta X}{\Delta Y}$

$X_j = X_i + \Delta X = X_i + L_{ij} * \sin Z_{ij}$

$Y_j = Y_i + \Delta Y = Y_i + L_{ij} * \cos Z_{ij}$

Figure (3-3): Coordinates Computations Of Traverse Points

7- Find the closure error for the X & Y coordinates. Since the coordinates of last control point of the traverse are known, then

closure error for X = εx = $(X_n)_{computed} - (X_n)_{known}$ (3.7)

closure error for Y = εy = $(Y_n)_{computed} - (Y_n)_{known}$ (3.8)

The linear error of closure is computd as follows

$$\varepsilon = (\varepsilon x^2 + \varepsilon y^2)^{0.5} \qquad (3.8)$$

8- Compute final adjusted coordinates of the PIs. The final coordinates of any point is computed using the following two equations :

$$(X_j)_{final} = (X_j)_{preliminary} - \frac{l}{L} * \varepsilon x \qquad (3.10)$$

$$(Y_j)_{final} = (Y_j)_{preliminary} - \frac{l}{L} * \varepsilon y \qquad (3.11)$$

Where

l : Cumulative length of courses up to point j .

L : Total length of all courses from start to end of the traverse .

9- Compute final length and azimuth of all courses using the final adjusted coordinates (see Fig. 3-3).

$$\Delta X = | X_j - X_i |$$

$$\Delta Y = | Y_j - Y_i |$$

$$L = (\Delta X^2 + \Delta Y^2)^{0.5} \quad (3.12)$$

$$Z = \tan^{-1} \frac{\Delta X}{\Delta Y}$$

To determine the quadrant of the course, any of the following cases may be choosen:

a- If $\Delta X > 0$ and $\Delta Y > 0$ then Azimuth = Z

b- If $\Delta X > 0$ and $\Delta Y < 0$ then Azimuth = 180 - Z

c- If $\Delta X < 0$ and $\Delta Y > 0$ then Azimuth = 360 - Z

d- If $\Delta X < 0$ and $\Delta Y < 0$ then Azimuth = 180 + Z

10- Compute the deflection angle at each PI (see Fig. 3-4).

$$\text{Deflection angle} = \Delta = | Z_j - Z_i | \quad (3.13)$$

Where

Z_j : Azimuth of forward tangent at PI.

Z_i : Azimuth of backward tangent at PI.

If the deflection angle is greater than 180 , then

$$\Delta_{final} = 360 - \Delta$$

Having determined the final absolute coordinates of all PIs and the azimuth of all courses, then it can be proceeded to the design of horizontal curves using the established coordinates. In the following sections, mathematical models and procedures will be developed for design of all the types of horizontal curves mentioned earlier, using coordinates method.

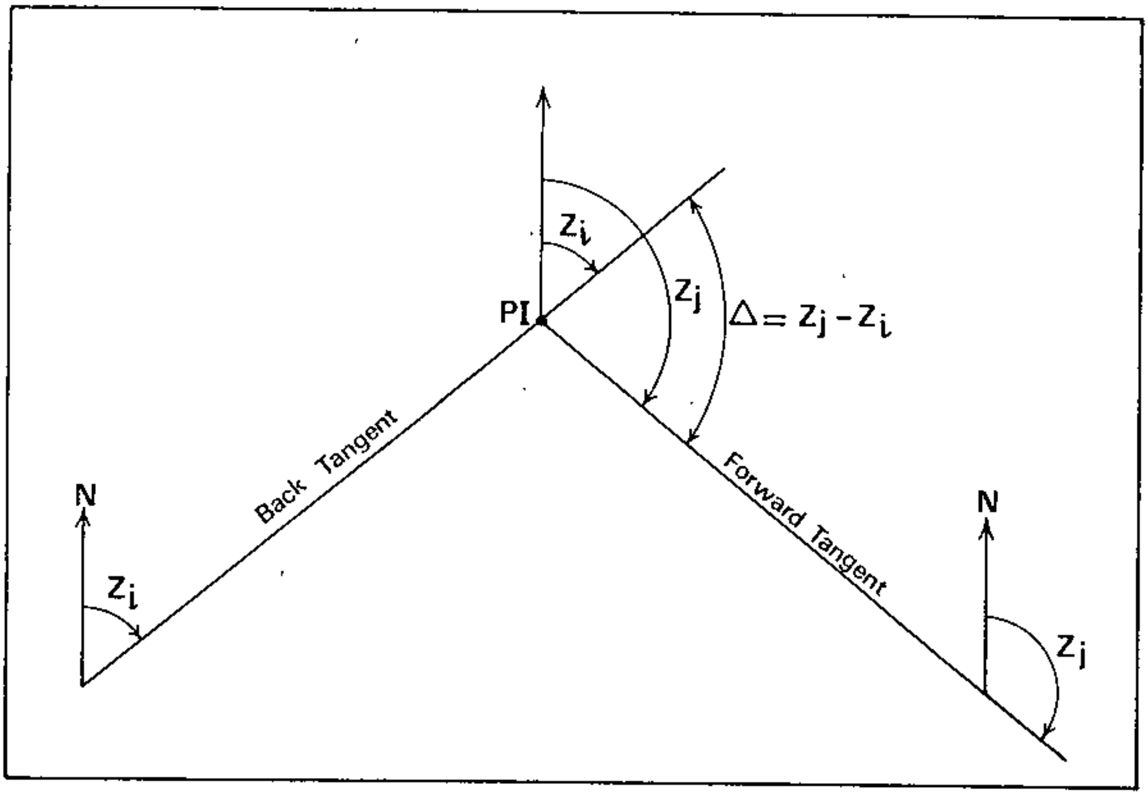


Figure (3-4): Showing How To Compute Deflection Angle At PI

3-3 SIMPLE CIRCULAR CURVE

Given :-

a- X & Y coordinates of PI (X_{PI}, Y_{PI}).

b- Azimuth of back & forward tangent (Az_1, Az_2).

c- Deflection angle Δ, at the PI.

d- Radius or degree of curve for the circular arc.

Procedure (see Fig. 3-5):

1- Compute tangent length,

$$T = R \tan \Delta/2 \qquad (3.14)$$

2- Compute external distance,

$$E = R (\sec \Delta/2 - 1) \qquad (3.15)$$

3- Compute the lengths of circular arc and long chord,

$$L_c = \pi R \Delta/180$$

$$\text{Long chord} = 2 R \sin \Delta/2$$

4- Compute X & Y coordinates of PC & PT, using eqs (3.5) & (3.6),

$$X_{PC} = X_{PI} + T \sin (Az_1 + 180)$$

$$Y_{PC} = Y_{PI} + T \cos (Az_1 + 180)$$

$$X_{PT} = X_{PI} + T \sin (Az_2)$$

$$Y_{PT} = Y_{PI} + T * \cos (Az_2)$$

5- Compute the X and Y coordinates for a group of points on the circular arc. The arc is divided into equal parts so that, c = R/20 , where c is the length of each part. In

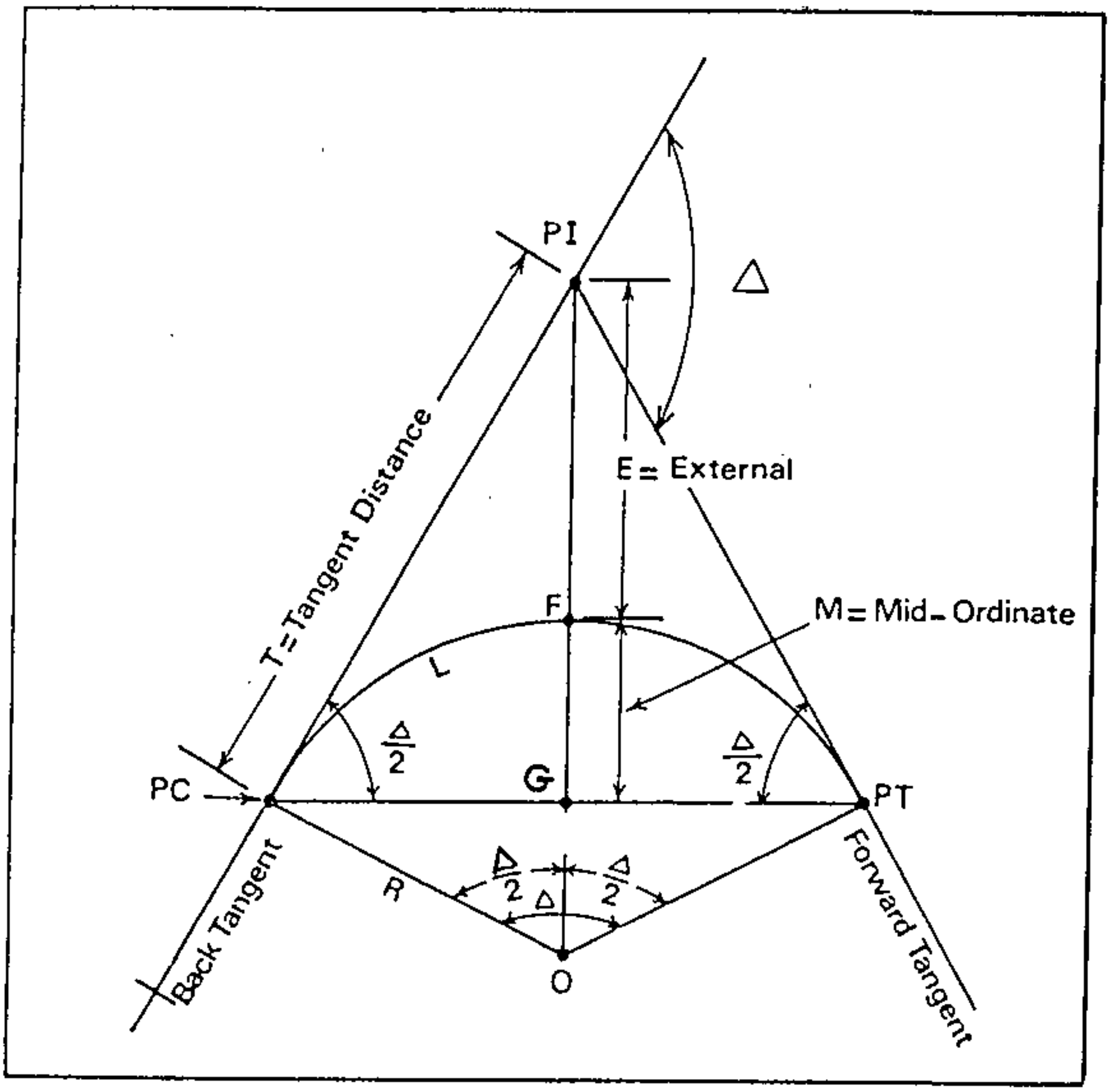


Figure (3-5): Simple Circular Curve [1]

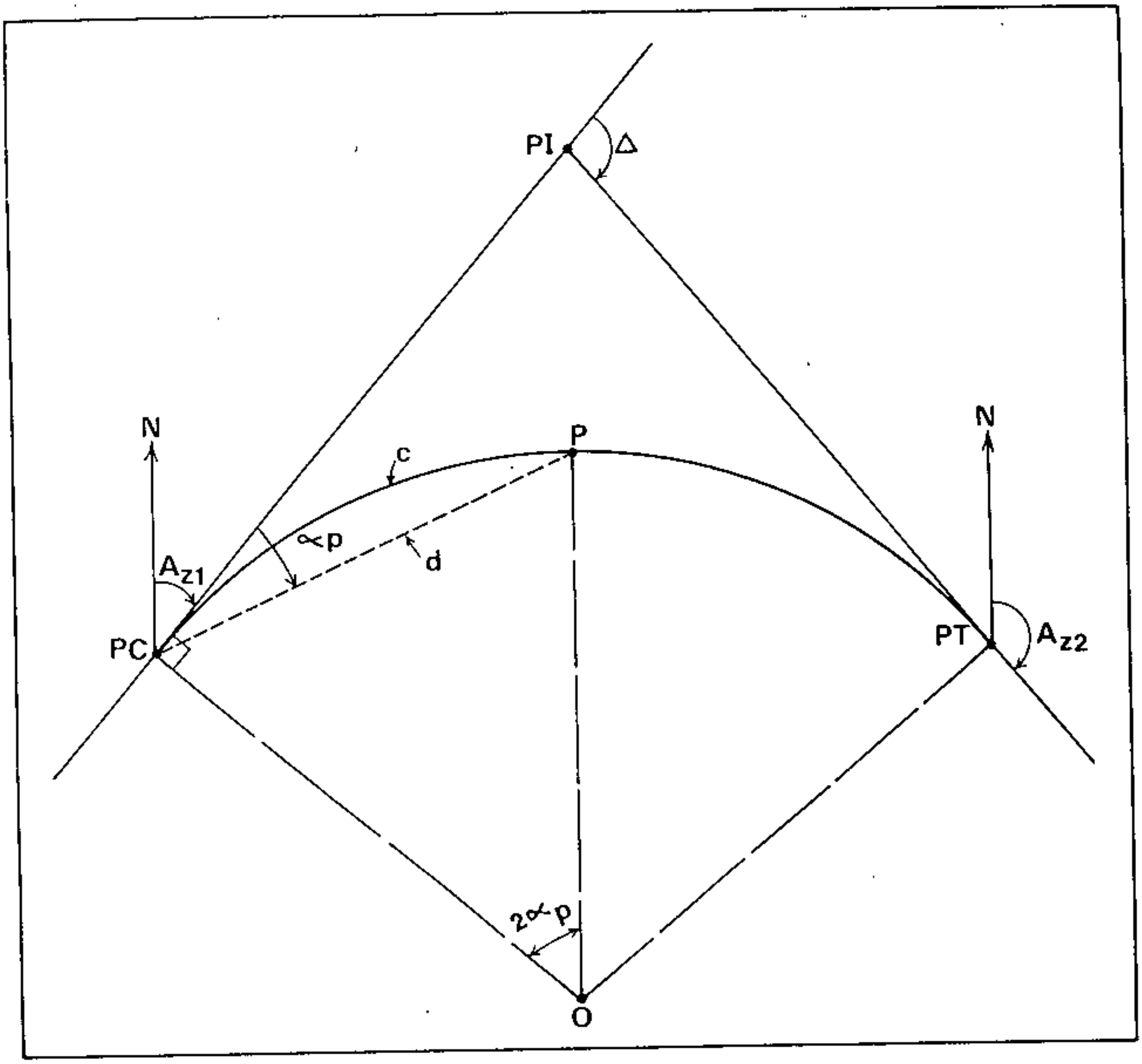


Figure (3-6): Showing How To Compute Coordinates For Points On The Simple Circular Curve this case, $l_1 = c$, $l_2 = 2c$, $l_3 = 3c$,..........$l_n = L_c$.

a- For any point P, find deflection angle α (Fig. 3-6),

$$\alpha_p = \frac{l_p}{2R} \quad \frac{180}{\pi} \qquad (3.17)$$

b- Compute the chord length from PC to P,

$$d = 2\ R \sin \alpha_p$$

c- Compute azimuth of line PC—P (see Fig. 3-6),

$$Az = Az_1 + \alpha_p$$

d- Compute absolute coordinates of point P,

$$X_p = X_{pc} + d \sin (Az_1 + \alpha_p)$$

$$Y_p = Y_{pc} + d \cos (Az_1 + \alpha_p)$$

6- Compute the coordinates of center point O,

$$X_o = X_{pc} + R \sin (Az_1 + 90)$$

$$Y_o = Y_{pc} + R \cos (Az_1 + 90)$$

7- Find the mid-ordinate distance [1],

$$M = R - R \cos \Delta/2 \qquad (3.18)$$

3-4 SPIRALED CIRCULAR CURVES

3-4-1 Equal-Tangent Spiraled Circular Curve

Given:

a- The X & Y coordinates of PI (X_{PI}, Y_{PI}).

b- Azimuths of back & forward tangents (Az_1 & Az_2).

c- Deflection angle Δ, at the PI.

d- Radius of circular curve R, or its degree of curve.

e- Length of spiral curve L_s, if not given then the design speed and rate of change of radial acceleration will be given.

Procedure (see Fig. 2-7):

1- Compute Tangent length using equation (2.20),

$$T = (R+S) \tan \frac{\Delta}{2} + X_o$$

Where:

$$X_o = X - R \sin \phi$$

$$S = Y - R (1 - \cos \phi)$$

$$\phi = L_s / 2 R$$

$$X = L_s \left(1 - \frac{\phi^2}{10} + \frac{\phi^4}{216} - \ldots\ldots \right)$$

$$Y = L_s \left(\frac{\phi}{3} - \frac{\phi^3}{42} + \frac{\phi^5}{1320} \ldots\ldots \right)$$

2- Compute length of circular arc,

$$L_c = \pi R (\Delta - 2\phi) 1/180$$

3- Compute External distance using equation (2.21),

$$E_s = (R+S) (\sec \Delta/2 - 1)+S$$

4- Find deflection angle for any point P on the left spiral at a distance l_p from TS (Fig. 3-7 & 3-8).

a- Find the spiral angle θ, for each point on the spiral using equation 2.13 (See Fig. 3-7),

$$\theta_p = (l_p/L_s)^2 * \phi$$

b- Find x_p and y_p from equations (2.14) & (2.15)

$$x_p = l \left(1 - \frac{\theta^2}{10} + \frac{\theta^4}{216} - \ldots\ldots. \right)$$

$$y_p = l \left(\frac{\theta}{3} - \frac{\theta^3}{42} + \frac{\theta^5}{1320} - \ldots\ldots.. \right)$$

c- Find the length of chord from TS to point P (See Fig. 3-8),

$$S_p = (x_p^2 + y_p^2)^{0.5}$$

d- Find deflection angle from back tangent (Fig. 3-8),

$$\alpha_p = \tan^{-1}(y_p/x_p)$$

5- Compute absolute coordinates of points on left spiral.

a- Find coordinates of point TS using eqs (3.5) & (3.6),

$$X_{TS} = X_{PI} + T \sin (A_{z1}+180)$$

$$Y_{TS} = Y_{PI} + T \cos (A_{z1}+180)$$

b- For any point P on the left spiral at a distance l_p from TS, find the azimuth of line TS--P (Fig. 3-9),

$$A_z \text{ of TS--P} = A_{z1} + \alpha_p$$

c- Compute absolute coordinates of point P,

$$X_P = X_{TS} + S_p \sin (A_{z1} + \alpha_p)$$

$$Y_P = Y_{TS} + S_p \cos (A_{z1} + \alpha_P)$$

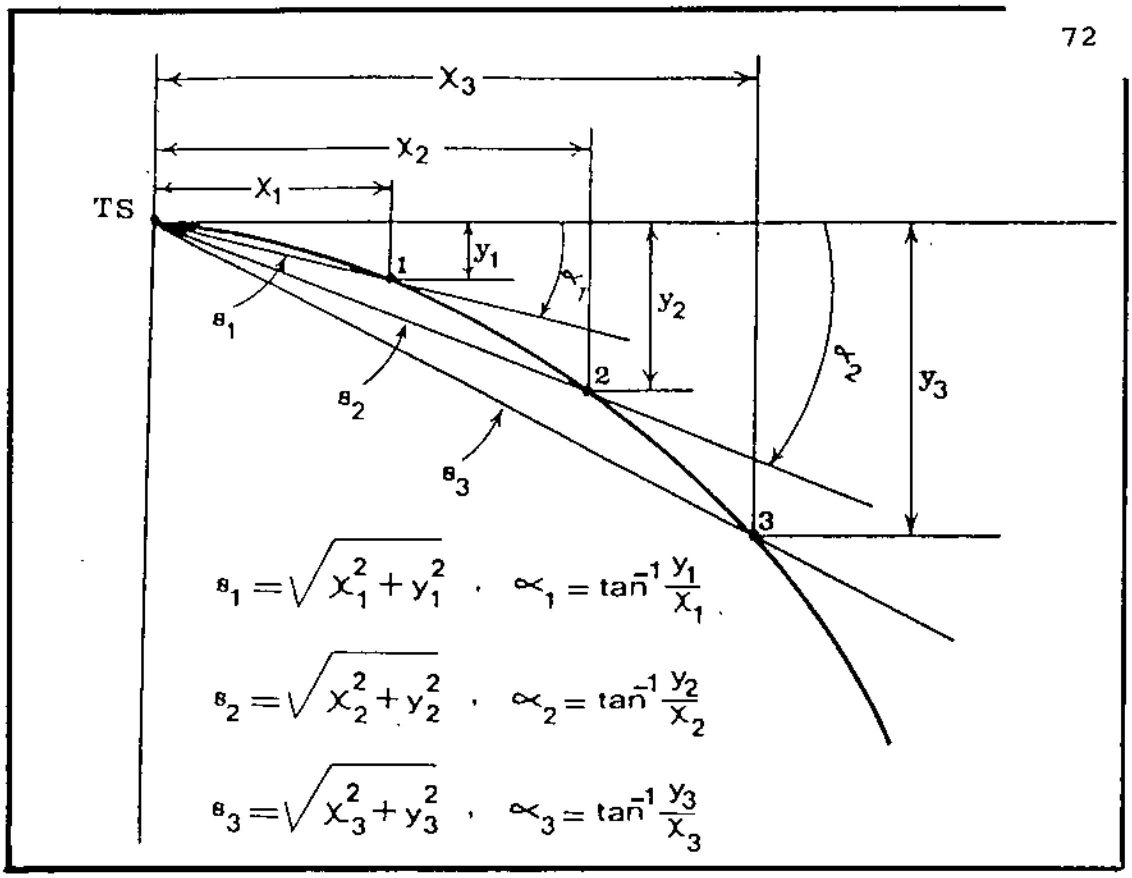


Figure (3-7): Deflection Angles For Points On The Left Spiral Of The Equal-Tangent Spiraled Circular Curve

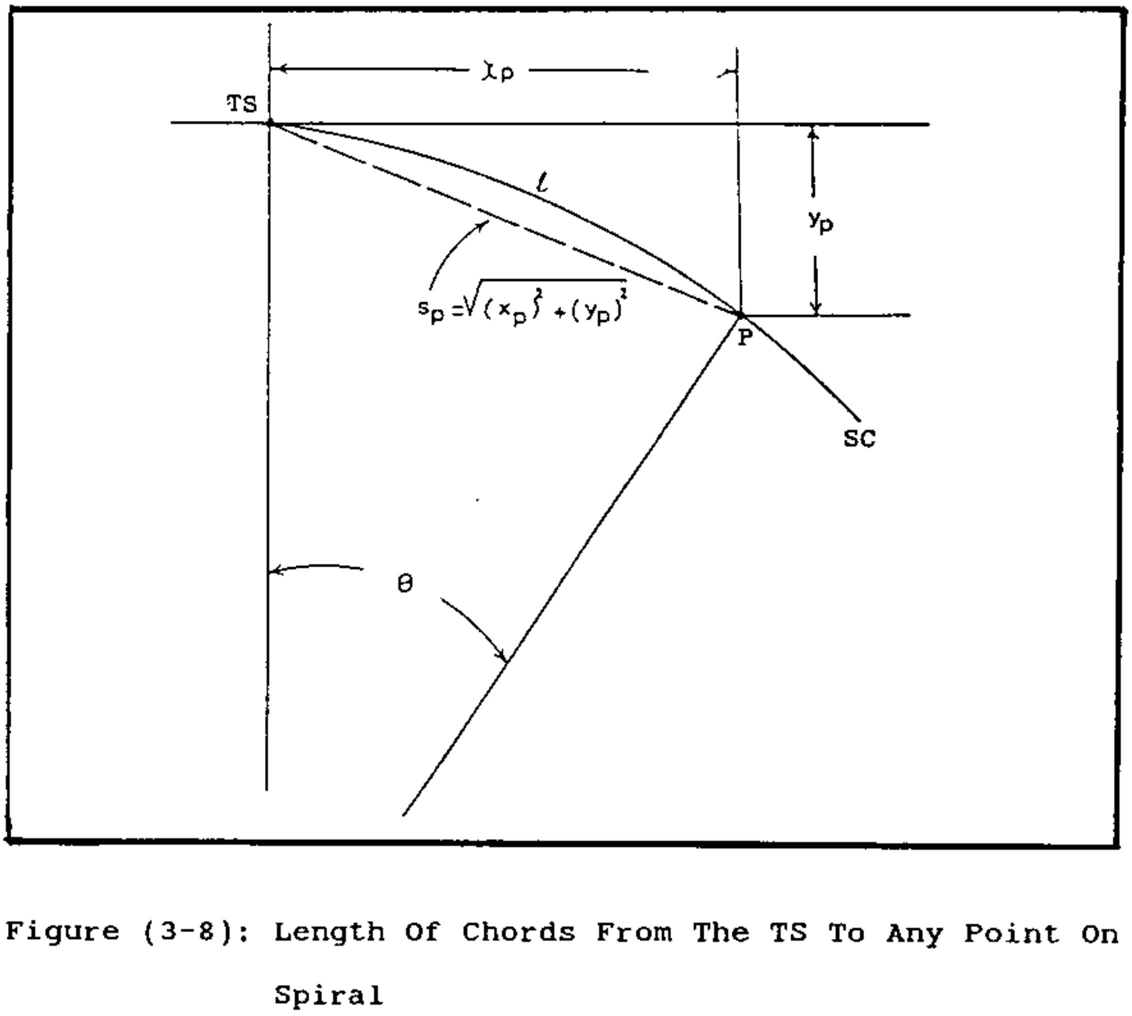


Figure (3-8): Length Of Chords From The TS To Any Point On Spiral

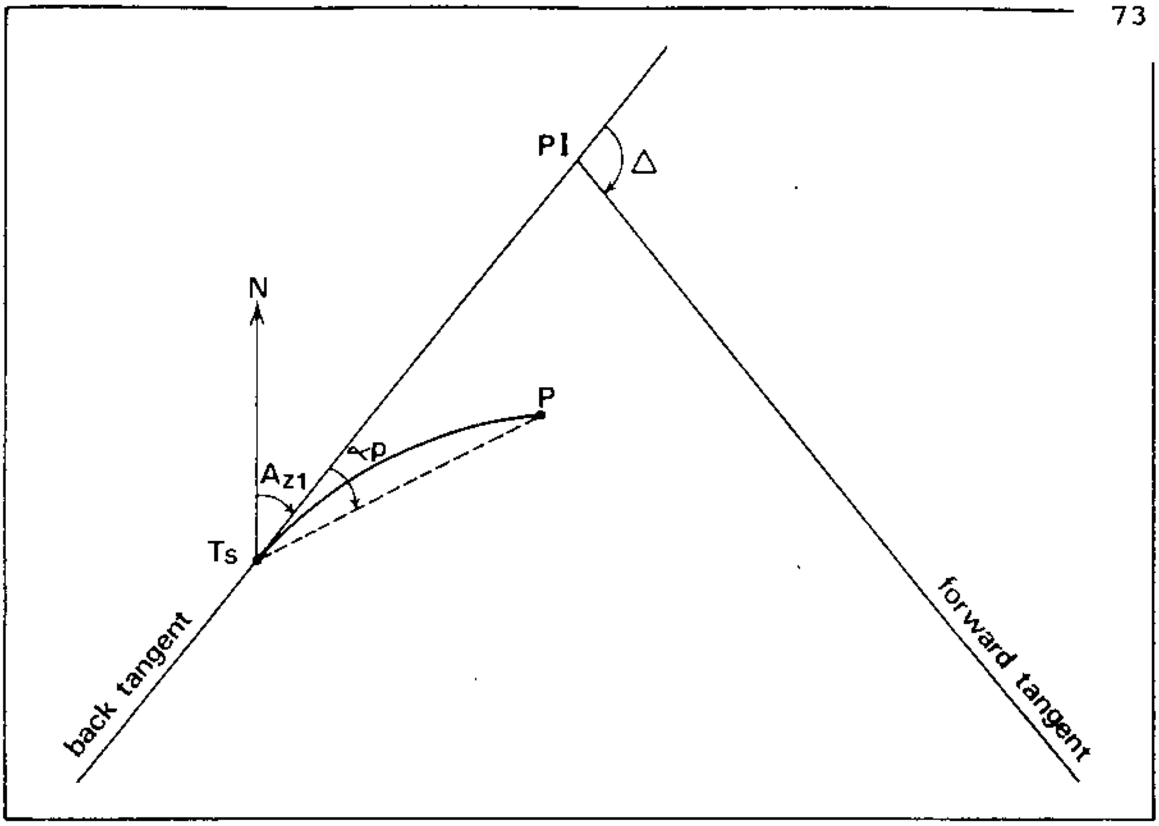


Figure (3-9): Showing How To Compute Coordinates For Points On Left Spiral Of Equal-Tangent Spiraled Circular Curve

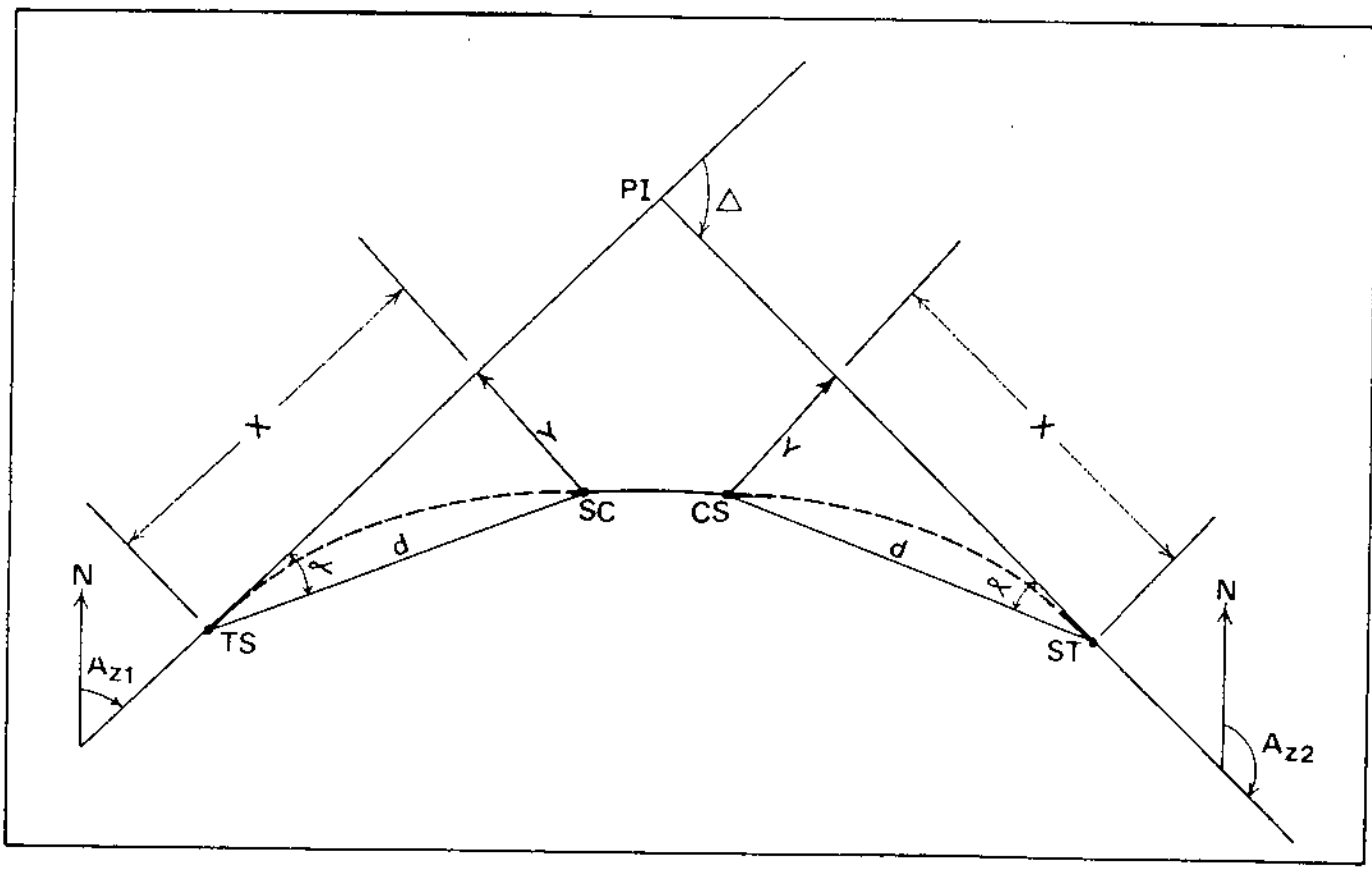


Figure (3-10): Showing How To Compute Coordinates Of Points SC & CS Of The Spiraled Circular Curves

6- Compute absolute coordinates of point SC.

a- Compute X & Y parameters for point SC as mentioned in step 1 (see Fig. 3-10).

b- Find the length of chord TS-SC.

$$d = (X^2 + Y^2)^{0.5}$$

c- Compute the deflection angle α

$$\alpha = \tan^{-1} Y/X$$

d- Compute final absolute coordinates of point SC.

$$X_{SC} = X_{TS} + d \sin (Az_1 + \alpha)$$

$$Y_{SC} = Y_{TS} + d \cos (Az_1 + \alpha)$$

7- Compute absolute coordinates for a group of points on the circular arc (Fig. 3-11),

a- Determine azimuth of tangent at SC,

Az of tangent at SC = $Az_1 + \phi$

b- For any point P, on the circular arc that is located at a distance l from SC,

$$\alpha = \frac{90}{\pi} \frac{l}{R}$$

c- Compute the length of chord SC – P,

$$d = 2 R \sin \alpha$$

d- Compute Azimuth of chord SC – P (see Fig. 3-11),

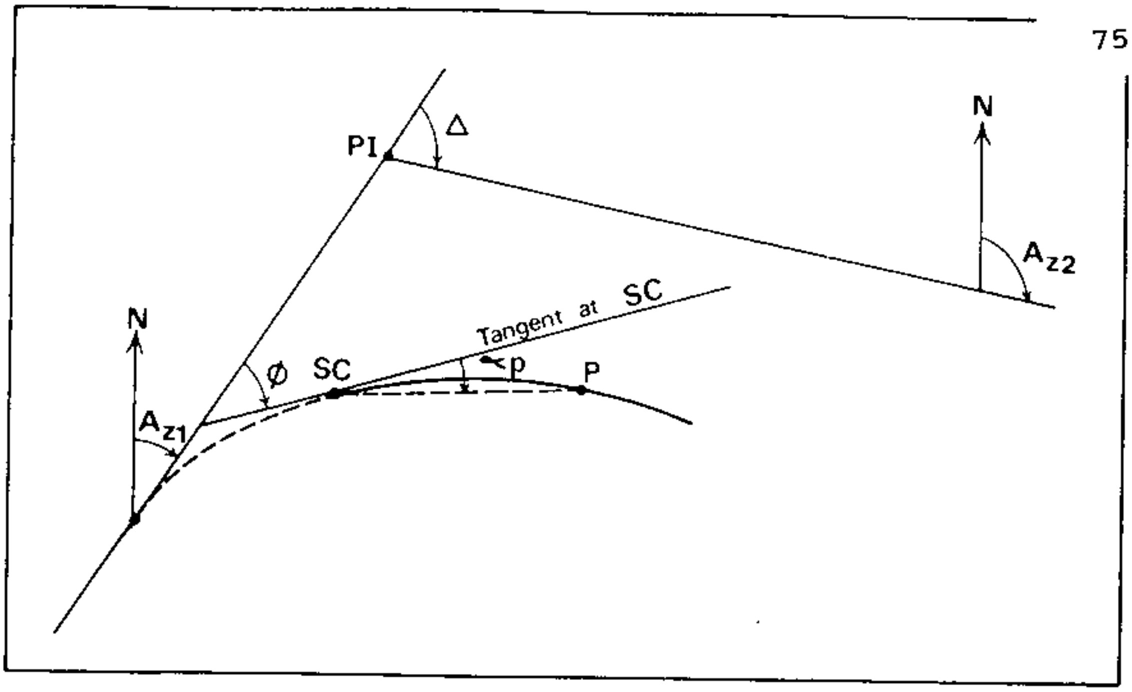


Figure (3-11): Showing How To Compute Coordinates Of Points On Circular Arc Of Equal-Tangent Spiraled Circular Curve

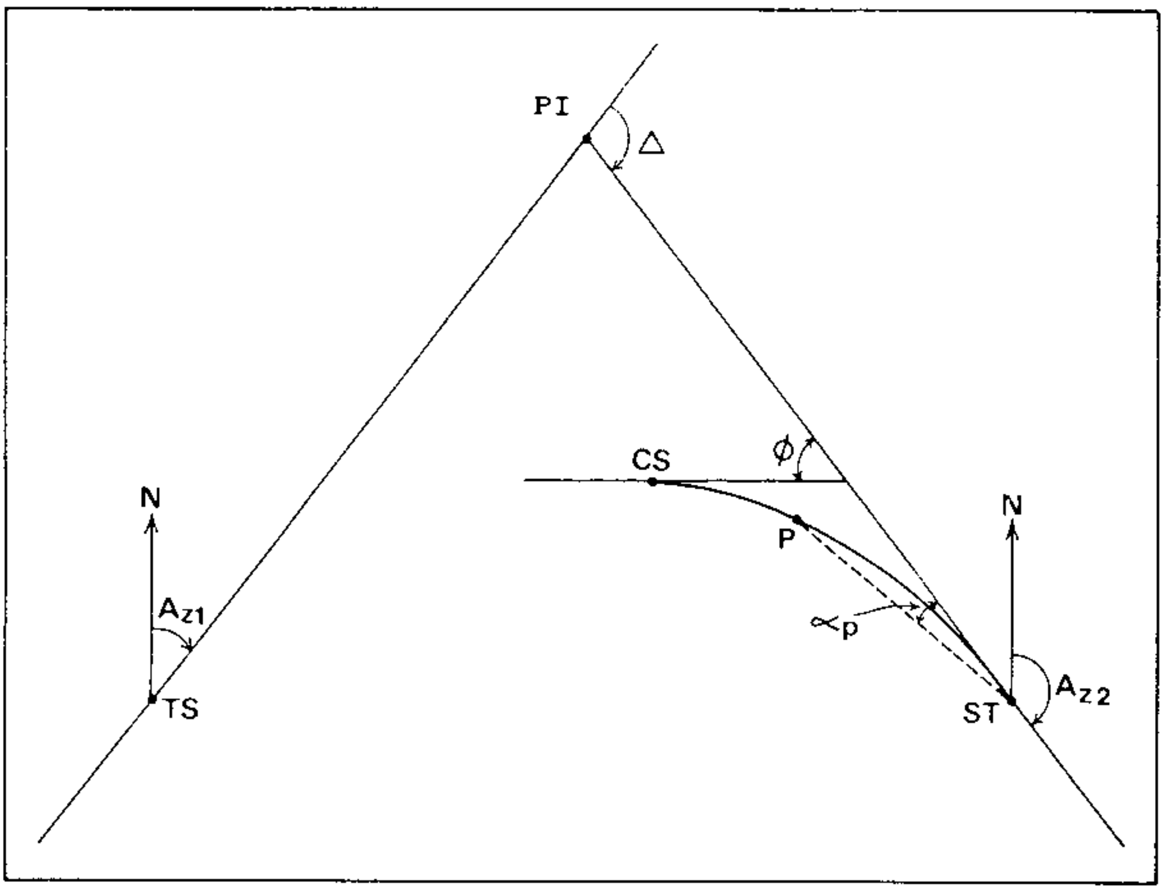


Figure (3-12): Showing How To Compute Coordinates Of Points On Right Spiral Of Equal-Tangent Spiraled Circular Curve

$$A_z = A_{z1} + \phi + \alpha$$

e- Determine absolute coordinates of point P,

$$X_P = X_{SC} + d \sin (A_{z1} + \phi + \alpha)$$

$$Y_P = Y_{SC} + d \cos (A_{z1} + \phi + \alpha)$$

8- Compute coordinates of point ST,

$$X_{ST} = X_{PI} + T \sin (A_{z2})$$

$$Y_{ST} = Y_{PI} + T \cos (A_{z2})$$

9- Compute coordinates for a group of points on the right spiral using the same procedure employed for the left spiral (See Fig. 3-12).

10- Using the computed absolute coordinates, find azimuth and distance of the line joining the TS to any point on the curve,

$$d = (\Delta X^2 + \Delta Y^2)^{0.5}$$

$$\text{Azimuth} = \tan^{-1} \frac{\Delta X}{\Delta Y}$$

3-4-2 Unequal-tangent spiraled circular curve

Given :

a- X & Y coordinates of PI (X_{PI}, Y_{PI}).

b- Azimuths of back and forward tangents at PI (A_{z1} & A_{z2}).

c- Deflection angle Δ, at the PI.

d- Radius of circular curve R, or its degree of curve.

e- Length of left and right spirals, L_{s1} & L_{s2} .

Procedure (see Fig. 2-8) :

1- Compute tangents lengths

a- Compute the spirals central angles,

$$\phi_1 = L_{s1}/2R \quad , \quad \phi_2 = L_{s2}/2R$$

b- Compute parameters X_1, Y_1, X_2, and Y_2 using equations (2.16) and (2.17) as shown below :

$$X_1 = L_{s1}\left(1 - \frac{\phi_1^2}{10} + \frac{\phi_1^4}{216} \ldots\ldots\right)$$

$$Y_1 = L_{s1}\left(\frac{\phi_1}{3} - \frac{\phi_1^3}{42} + \frac{\phi_1^5}{1320} - \ldots\ldots\right)$$

$$X_2 = L_{s2}\left(1 - \frac{\phi_2^2}{10} + \frac{\phi_2^4}{216} \ldots\ldots\right)$$

$$Y_2 = L_{s2}\left(\frac{\phi_2}{3} - \frac{\phi_2^3}{42} + \frac{\phi_2^5}{1320} - \ldots\ldots\right)$$

c- Compute the abscissas X_{o1} , X_{o2} and shifts S_1 and S_2 using equations (2.18) and (2.19) as shown below :

$$X_{o1} = X_1 - R \sin \phi_1$$

$$X_{o2} = X_2 - R \sin \phi_2$$

$$S_1 = Y_1 - R\ (1-\cos \phi_1)$$

$$S_2 = Y_2 - R\ (1-\cos \phi_2)$$

d- The tangents lengths are then determined using equations (2.18) and (2.19) :

$$T_1 = X_{o1} + (R + S_1) \tan \Delta/2 + \frac{(S_2 - S_1)}{\sin \Delta}$$

$$T_2 = X_{o2} + (R + S_2) \tan \Delta/2 - \frac{(S_2 - S_1)}{\sin \Delta}$$

2- Compute the length of circular arc,

$$L_c = \pi R (\Delta - \phi_1 - \phi_2) / 180$$

3- Compute the absolute coordinates of main points TS,SC,CS and ST.

a- Point TS

$$X_{TS} = X_{PI} + T_1 \sin (Az_1 + 180)$$

$$Y_{TS} = Y_{PI} + T_1 \cos (Az_1 + 180)$$

b- Point SC

1- $d = (X_1^2 + Y_1^2)^{0.5}$

2- $\alpha = \tan^{-1} Y_1/X_1$

3- $X_{SC} = X_{TS} + d \sin (Az_1 + \alpha)$

$Y_{SC} = Y_{TS} + d \cos (Az_1 + \alpha)$

c- Point ST

$$Y_{ST} = X_{PI} + T_2 \sin (Az_2)$$

$$Y_{ST} = Y_{PI} + T_2 \cos (Az_2)$$

d- Point CS (See Fig. 3-13)

1- find length of chord SC – CS

$$D = 2\ R \sin\ (\Delta - \phi_1 - \phi_2)/2$$

2- deflection angle $\alpha = (\Delta - \phi_1 - \phi_2)/2$

3- find azimuth of tangent at SC,

$$Az = Az_1 + \phi_1$$

4- $X_{CS} = X_{SC} + D \sin\ (Az_1 + \phi_1 + \alpha)$

$Y_{CS} = Y_{SC} + D \cos\ (Az_1 + \phi_1 + \alpha)$

4- Compute coordinates for a group of points on left spiral, circular arc and right spiral as described earlier, in the case of equal-tangent spiraled circular curve.

5- After computing coordinates of all points, find the azimuth and distance of the line joining TS to any point on the curve,

$$d = (\Delta X^2 + \Delta Y^2)^{0.5}$$

$$Azimuth = \tan^{-1} \frac{\Delta X}{\Delta Y}$$

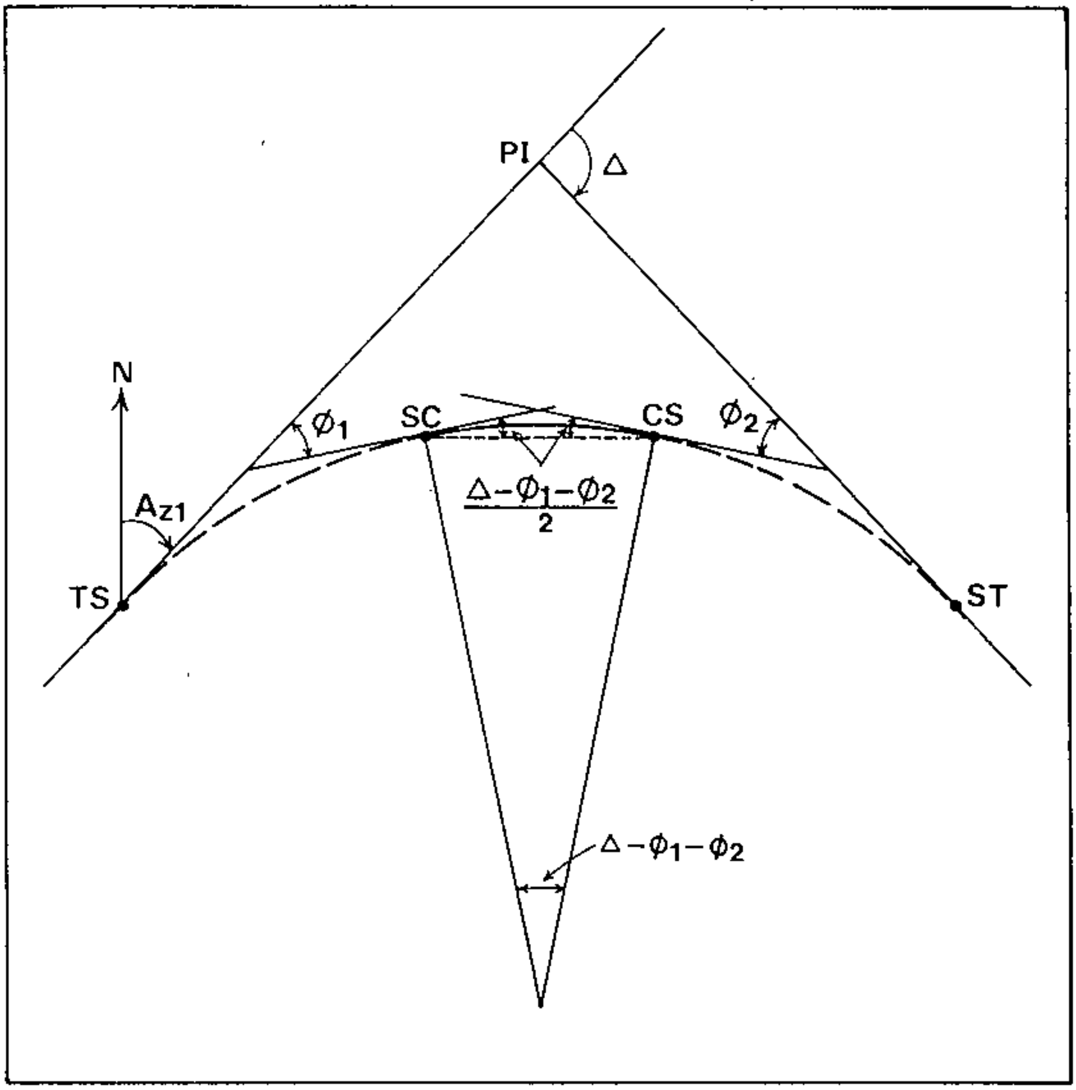


Figure (3-13): Showing How To Compute Coordinates Of Point CS Of The Unequal-Tangent Spiraled Circular Curve

3-4-3 Spiral applied to existing circular curve

Given:-

a- X & Y coordinates of PI (X_{PI}, Y_{PI}).

b- Azimuth of back & forward tangents (A_{z1} & A_{z2}).

c- Deflection angle Δ, at the PI.

d- Radius of circular curve or its degree of curve

e- Spiral central angle ϕ.

Procedure: (see Figures 2-11 & 2-12)

1- Find length of spiral as shown in eq (2.29),

$$L_s = \frac{R(1 - \cos \phi)}{y_1}$$

where y_1 is the offset distance of point SC for a spiral of unit length.

2- Find the tangent distance,

$$T = A + R \tan \Delta/2$$

Where

$$A = X - R \sin \phi$$

$$X = L_s \, x_1$$

x_1 is the tangent distance of point SC for a spiral of unit length .

3- Find length of remaining circular arc,

$$L_c = \pi \, (\Delta - 2\phi) \, R/180$$

4- Compute coordinates of main points TS, SC, CS, ST as described for equal-tangent spiraled circular curve.

6- Find the azimuth and distance of the line joining TS to any point on the curve.

3-5 DOUBLE SPIRAL CURVES

3-5-1 Equal-tangent double spiral curve

Given :

a- Absolute coordinates of PI (X_{PI}, Y_{PI}).

b- Azimuth of back and forward tangents at PI (Az_1 & Az_2).

c- Deflection angle Δ, at the PI.

d- Length of spiral L_s.

Procedure (see Fig. 2-9):

1- Find the tangent length using equation (2.25):

$$T = X + Y \tan \frac{\Delta}{2}$$

Where

$$X = L_s \left(1 - \frac{\phi^2}{10} + \frac{\phi^4}{216} - \ldots\ldots. \right)$$

$$Y = L_s \left(\frac{\phi}{3} - \frac{\phi^3}{42} + \frac{\phi^5}{1320} - \ldots.. \right)$$

$$\phi = \frac{\Delta}{2}$$

2- Find external distance using equation (2.26),

$$E_s = \frac{Y}{\cos \Delta/2}$$

3- Find coordinates of point TS using eqs (3.5) and (3.6),

$$X_{TS} = X_{PI} + T \sin (A_{z1} + 180)$$

$$Y_{TS} = Y_{PI} + T \cos (A_{z1} + 180)$$

4- Find coordinates of common tangency point SS,

$$X_{SS} = X_{TS} + d \sin (A_{z1} + \alpha)$$

$$Y_{SS} = Y_{TS} + d \cos (A_{z1} + \alpha)$$

Where

$$d = (X^2 + Y^2)^{0.5}$$

$$\alpha = \tan^{-1} Y/X$$

5- Find coordinates of point ST,

$$X_{ST} = X_{PI} + T \sin (A_{z2})$$

$$Y_{ST} = Y_{PI} + T \cos (A_{z2})$$

6- Find coordinates for a group of points on left and right spirals as shown earlier in the case of equal-tangent spiraled circular curve.

7- Find the distance and azimuth of the line joining TS to any other point on the curve,

$$d = (\Delta X^2 + \Delta Y^2)^{0.5}$$

$$Az = \tan^{-1} \frac{\Delta X}{\Delta Y}$$

3-5-2 Unequal-Tangent Double Spiral Curve

Given :

a- The absolute coordinates of PI (X_{PI}, Y_{PI}).

b- Azimuth of back and forward tangents at PI (Az_1, Az_2).

c- Deflection angle Δ, at the PI.

d- Length of left or right spiral (L_{s1} or L_{s2}).

d- The left or right spiral central angle (ϕ_1 or ϕ_2).

Procedure (see Fig. 2-10):

1- Compute either ϕ_1 or ϕ_2 according to what given in the problem using the following relation:

$$\Delta = \phi_1 + \phi_2$$

2- Find the unknown spiral length as follows,

$$L_s = 2 \phi R$$

Where R is the radius at the common point SS, and is computed from the given ϕ and L_s of either spiral. ϕ is in radian.

3- Compute X_1, Y_1, X_2 and Y_2 using eqs (2.16) and (2.17) as shown below :

$$X_1 = L_{s1} \left(1 - \frac{\phi_1^2}{10} + \frac{\phi_1^4}{216} \ldots\ldots\right)$$

$$Y_1 = L_{s1}\left(\frac{\phi_1}{3} - \frac{\phi_1^3}{42} + \frac{\phi_1^5}{1320} - \ldots\ldots\right)$$

$$X_2 = L_{s2} \left(1 - \frac{\phi_2^2}{10} + \frac{\phi_2^4}{216} \ldots\ldots\right)$$

$$Y_2 = L_{s2}\left(\frac{\phi_2}{3} - \frac{\phi_2^3}{42} + \frac{\phi_2^5}{1320} - \ldots\ldots\right)$$

4- Compute tangent distances using eqs (2.27) and (2.28),

$$T_1 = t_1 + t_3$$

$$T_2 = t_2 + t_4$$

Where

$$t_3 = X_1 - Y_1/\tan \phi_1$$

$$t_4 = X_2 - Y_2/\tan \phi_2$$

$$t_1 = PN = NJ * \sin \phi_2/\sin \Delta$$

$$t_2 = PJ = NJ * \sin \phi_1/\sin \Delta$$

$$NJ = \frac{Y_1}{\sin \phi_1} + \frac{Y_2}{\sin \phi_2}$$

2- Compute coordinates of the main points TS, SS and ST (See Fig. 3-14),

a- point TS

$$X_{TS} = X_{PI} + T_1 \sin (A_{z1} + 180)$$

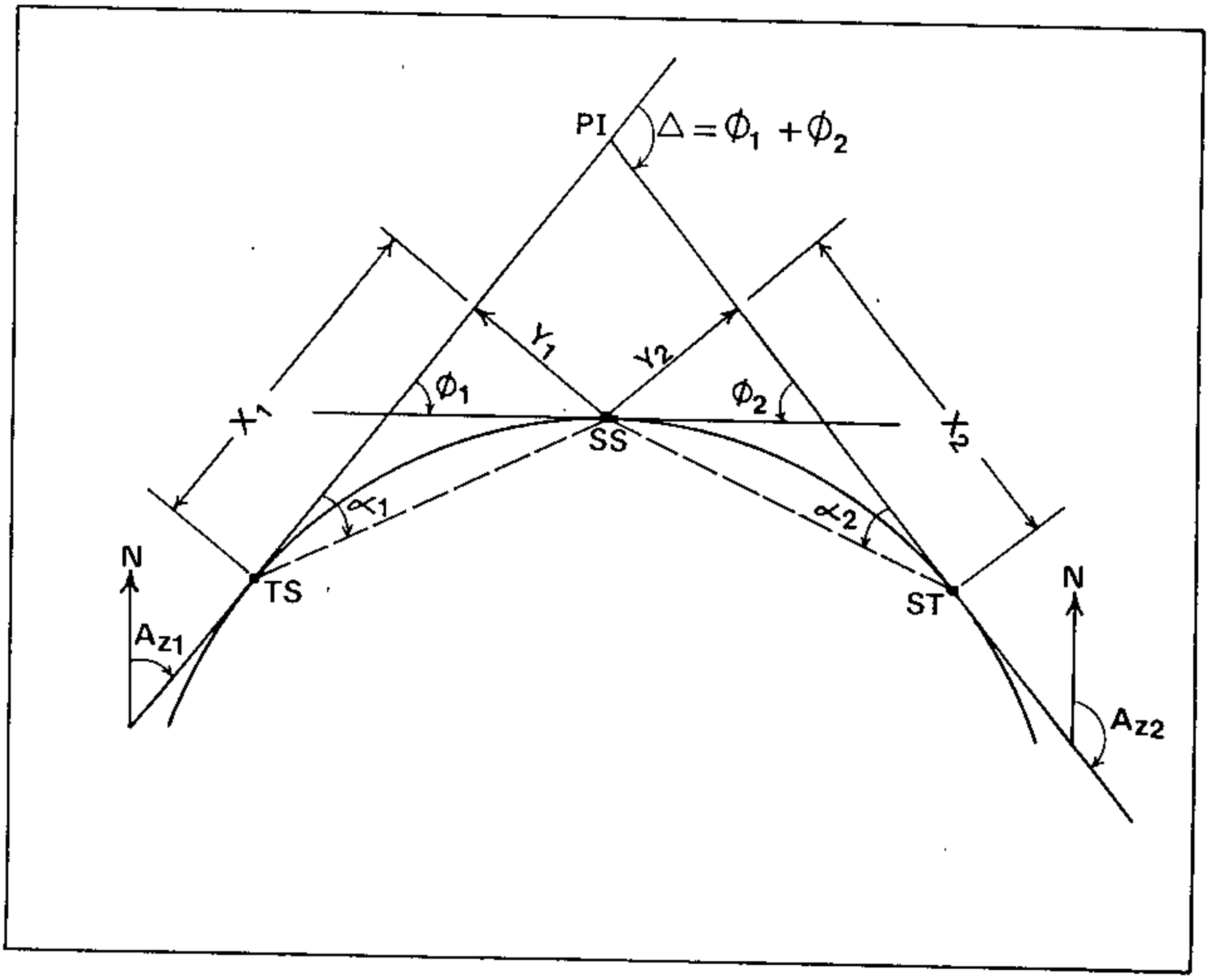


Figure (3-14): Showing How To Compute Coordinates For The Main Points Of Unequal - Tangent Double Spiral Curve

$$Y_{TS} = Y_{PI} + T_1 \cos (Az_1 + 180)$$

b- point SS

$$X_{SS} = X_{TS} + d_1 \sin (Az_1 + \alpha_1)$$

$$Y_{SS} = Y_{TS} + d_1 \cos (Az_1 + \alpha_1)$$

Where

$$d_1 = (X_1^2 + Y_1^2)^{0.5}$$

$$\alpha_1 = \tan^{-1} \frac{Y_1}{X_1}$$

c- Point ST

$$X_{ST} = X_{PI} + T_2 \sin (Az_2)$$

$$Y_{ST} = Y_{PI} + T_2 \cos (Az_2)$$

3- Find the coordinates for a group of points on the left and right spirals in the same manner described for the spiraled circular curve.

4- Find the distance and azimuth of the line that joins TS to any other point on the curve.

$$d = (\Delta X^2 + \Delta Y^2)^{0.5}$$

$$Az = \tan^{-1} \Delta X/\Delta Y$$

3-6 COMPOUND CURVES

3-6-1 Two-Centered Compound Curve

Given:-

a- Absolute coordinates of PI (X_{PI}, Y_{PI}).

b- Azimuth of back & forward tangents at PI (A_{z1} & A_{z2}).

c- Deflection angle Δ, at the PI.

d- Radii of left and right circular arcs, R_1 & R_2

e- Δ_1 or Δ_2

Procedure (see Fig. 3-15) :

1- Find tangents lengths

$$NJ = R_1 \tan \Delta_1/2 + R_2 \tan \Delta_2/2$$

$$PN = NJ \sin \Delta_2 / \sin \Delta$$

$$PJ = NJ \sin \Delta_1 / \sin \Delta$$

Therefore

$$T_1 = PN + R_1 \tan \Delta_1/2$$

$$T_2 = PJ + R_2 \tan \Delta_2/2$$

2- Find length of both circular arcs,

$$L_1 = \pi R_1 \Delta_1/180$$

$$L_2 = \pi R_2 \Delta_2/180$$

3- Find coordinates of point T_1 using eqs (3.5) and (3.6),

$$X_{PC} = X_{PI} + T_{s1} \sin (A_{z1} + 180)$$

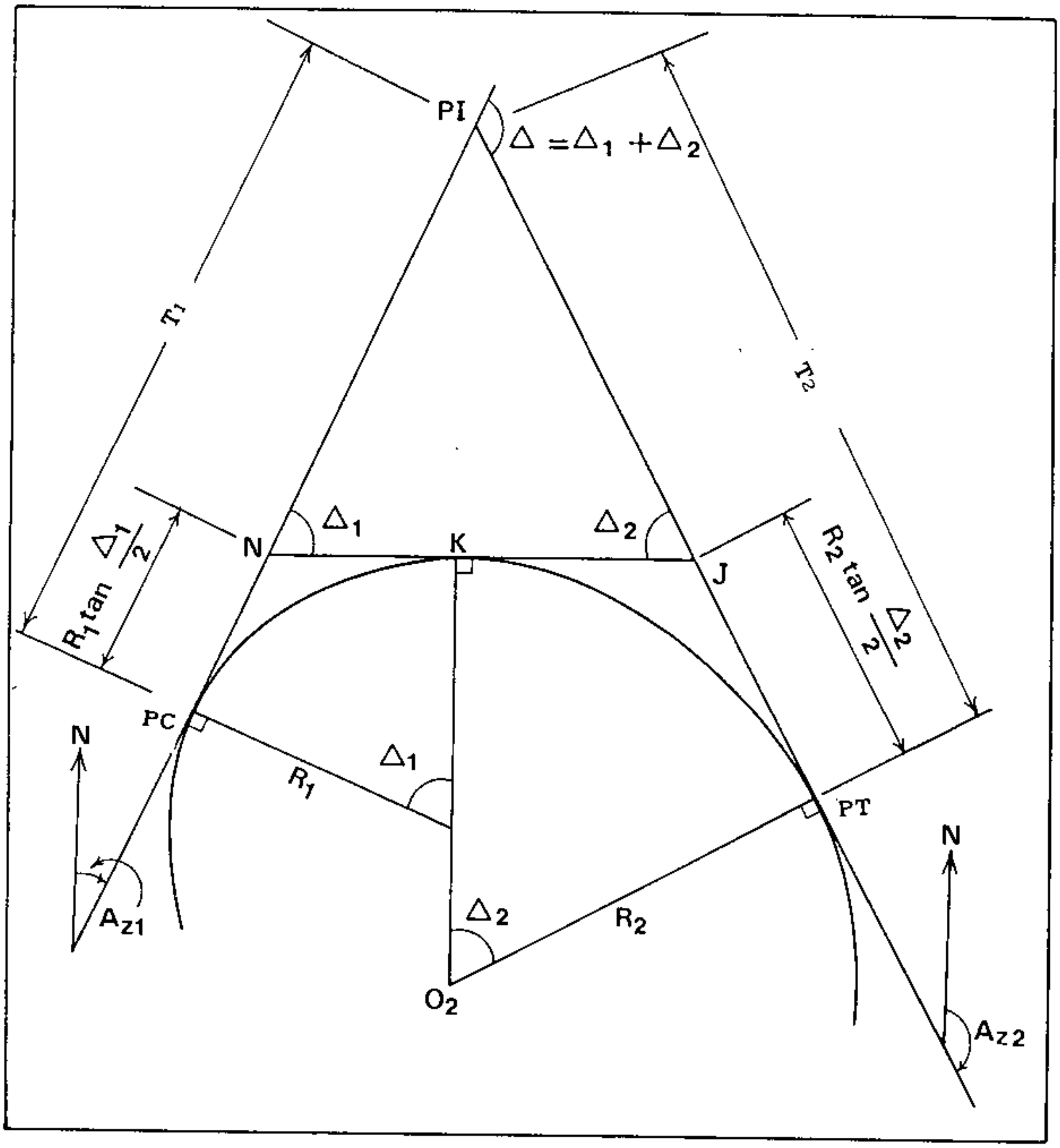


Figure (3-15): Two-Centered Compound Curve

$Y_{PC} = Y_{PI} + T_{s1} \cos (A_{z1} + 180)$

4- Find coordinates of common point K (Fig. 3-16),

The chord $T_1K = d = 2 R_1 \sin \Delta_1/2$

Azimuth of chord $T_1K = A_{z1} + \Delta_1/2$

Hence

$X_K = X_{T1} + d \sin (A_{z1} + \Delta_1/2)$

$Y_K = Y_{T1} + d \cos (A_{z1} + \Delta_1/2)$

5- Find coordinates of point PT

$X_{PT} = X_{PI} + T_{s2} \sin (A_{z2})$

$Y_{PT} = Y_{PI} + T_{s2} \cos (A_{z2})$

6- Find coordinates for a group of points on left circular arc as shown for the simple circular curve.

7- Establish azimuth of tangent at point K

Azimuth $= A_{z1} + \Delta_1$

8- Find coordinates for a group of points on right circular arc as follows:

a- For any point P a distance l_p from K,

$$\alpha_p = \frac{l_p}{2R_2} * \frac{180}{\pi}$$

$d = 2 R_2 \sin \alpha_p$

b- Coordinates of point P are then computed

$X_P = X_K + d \sin (A_{z1} + \Delta_1 + \alpha_p)$

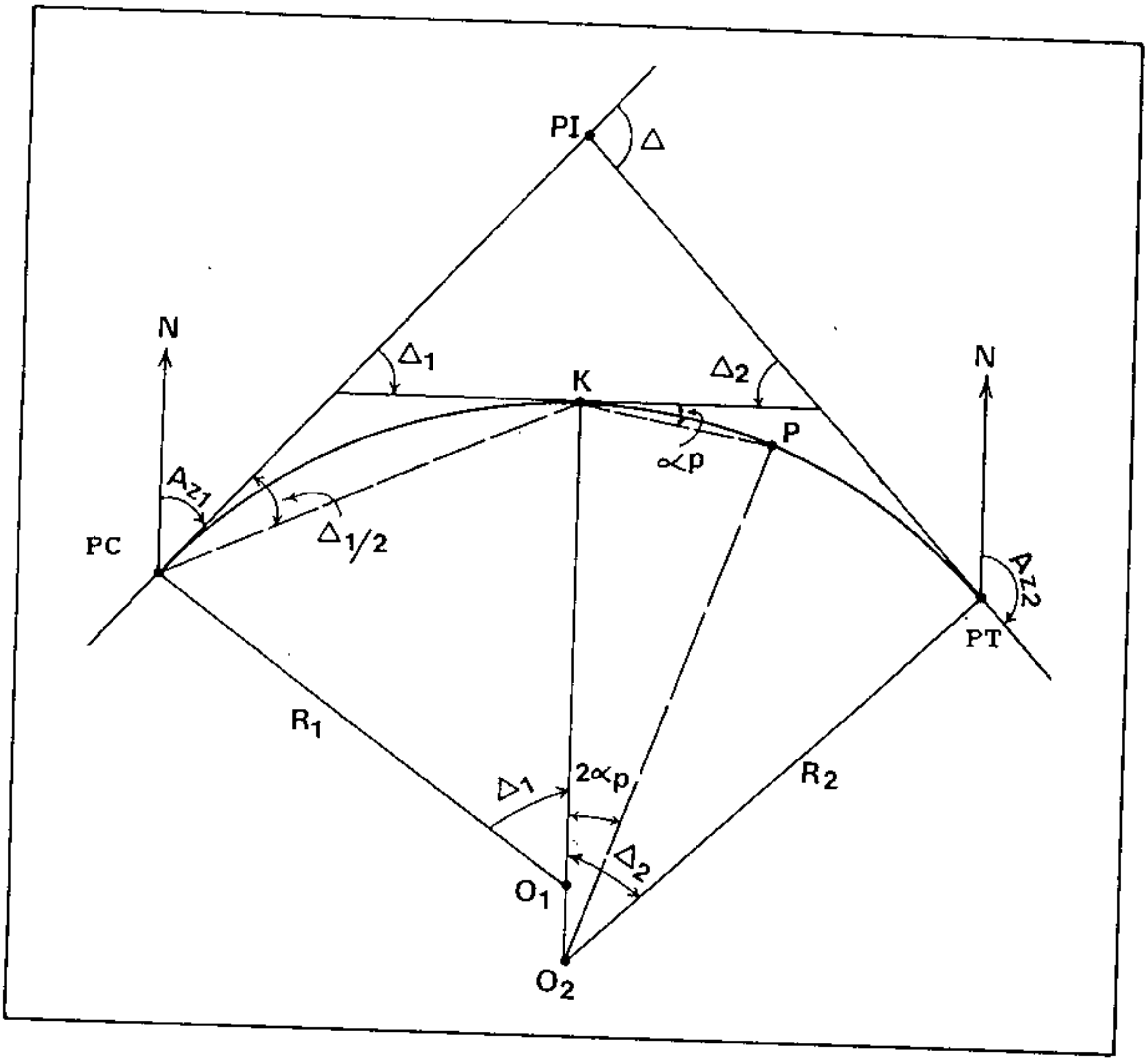


Figure (3-16): Showing How To Compute Coordinates For Points On The 2-Centered Compound Curve

$Y_P = Y_K + d \cos (A_{z1} + \Delta_1 + \alpha_p)$

9- Compute the coordinates of center points O_1 & O_2 for each arc,

$X_{O1} = X_{PC} + R_1 \sin (A_{z1} + 90)$

$Y_{O1} = Y_{PC} + R_1 \cos (A_{z1} + 90)$

Similarly

$X_{O2} = X_{PT} + R_2 \sin (A_{z2} + 90)$

$Y_{O2} = Y_{PT} + R_2 \cos (A_{z2} + 90)$

10- Find the azimuth and distance of the line joining the point T_1 to any other point on the compound curve[1,2,6]

3-6-2 Three-Centered Compound Curve

Given :

a- Absolute coordinates of PI (X_{PI},Y_{PI}).

b- Azimuth of back & forward tangents at PI (A_{z1} & A_{z2})

c- Deflection angle Δ, at the PI.

d- Radii R_1,R_2 and R_3.

e- Any pair of the 3 deflection angles Δ_1, Δ_2 and Δ_3.

Procedure (see Fig. 3-17) :

1- Find tangents lengths [1,2,5],

$NK = R_1 \tan \Delta_1/2 + R_2 \tan \Delta_2/2$

$KL = R_2 \tan \Delta_2/2 + R_3 \tan \Delta_3/2$

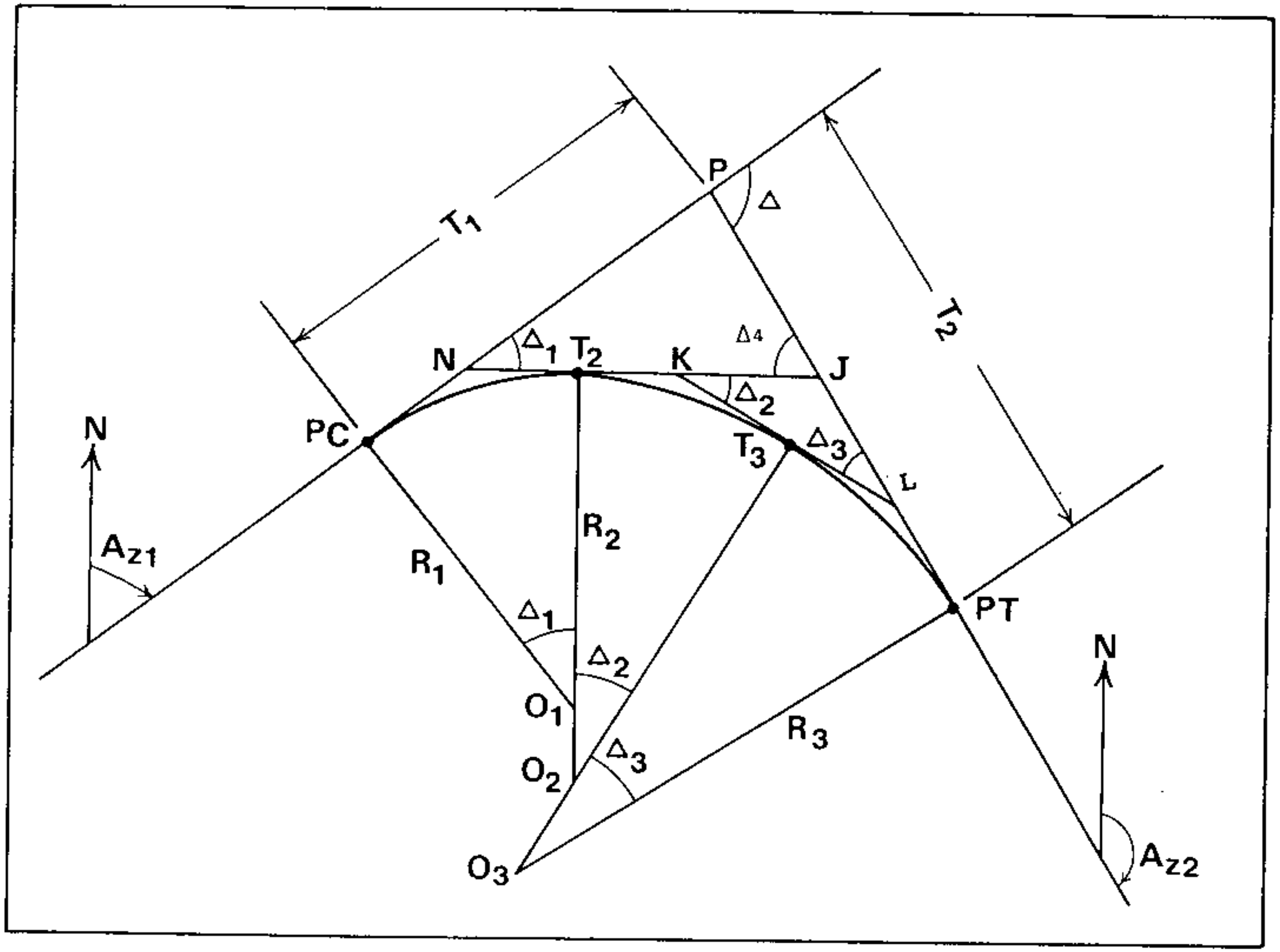


Figure (3-17): Three-Centered Compound Curve

$KJ = KL \sin \Delta_3 / \sin \Delta_4$

$JL = KL \sin \Delta_2 / \sin \Delta_4$

$NJ = NK + KJ$

By solving the triangle PNJ we obtain

$PN = NJ \sin \Delta_4 / \sin \Delta$

$PJ = NJ \sin \Delta_1 / \sin \Delta$

Therefore, the total tangents lengths are

$$T_1 = PN + R_1 \tan \Delta_1/2 \qquad (3.21)$$

$$T_2 = PJ + JL + R_3 \tan \Delta_3/2 \qquad (3.22)$$

2- Find the length of circular arcs,

$L_1 = \pi \; R_1 \; \Delta_1/180$

$L_2 = \pi \; R_2 \; \Delta_2/180$

$L_3 = \pi \; R_3 \; \Delta_3/180$

Where L_1, L_2 and L_3 are the lengths of left , middle and right arcs respectively.

3- Compute the absolute coordinates of main points PC,T1, T2 and PT using eqs (3.5) and (3.6):.

a- Point PC

$X_{PC} = X_{PI} + T_1 \sin (Az_1 + 180)$

$Y_{PC} = Y_{PI} + T_1 \cos (Az_1 + 180)$

b- Point T1

$X_{T1} = X_{PC} + d_1 \sin (Az_1 + \Delta_1/2)$

$Y_{T1} = Y_{PC} + d_1 \cos (A_{z1} + \Delta_1/2)$

Where

$d_1 = 2 R_1 \sin \Delta_1/2$

c- Point T2

$X_{T2} = X_{T1} + d_2 \sin (A_{z1} + \Delta_1 + \Delta_2/2)$

$Y_{T2} = Y_{T1} + d_2 \cos (A_{z1} + \Delta_1 + \Delta_2/2)$

Where

$d_2 = 2 R_2 \sin \Delta_2/2$

d- Point PT

$X_{PT} = X_{PI} + T_2 \sin (A_{z2})$

$Y_{PT} = Y_{PI} + T_2 \cos (A_{z2})$

4- Compute the coordinaes of center points of each arc as described for the 2-centred compound curve.

5- Establish the azimuth of tangents at T_2 and T_3,

Azimuth at T1 = $A_{z1} + \Delta_1$

Azimuth at T2 = $A_{z1} + \Delta_1 + \Delta_2$

6- Compute the absolute coordinates for a group of points on each arc, using the same procedure employed for the simple circular curve.

7- Use the computed absolute coordinates to determine the distance and azimuth of the line joining the point T_1 to any other point on the compound curve.

3-7 SPIRALED COMPOUND CURVES

3-7-1 Two-centered compound curve with 1 connecting spiral

Given :

a- Absolute coordinates of PI (X_{PI}, Y_{PI}).

b- Azimuth of back & forward tangents at PI (Az_1 & Az_2)

c- Deflection angle Δ, at the PI.

d- Radii R_1 and R_2 or degrees of curve D_1 and D_2.

e- Deflection angle Δ_1 or Δ_2.

f- Length of connecting spiral, L_s .

Procedure (see Fig. 2-13):

1- Compute spiral central angle for each half of the spiral

$$\phi_1 = \frac{L_s}{2R_1}$$

$$\phi_2 = \frac{L_s}{2R_2}$$

2- Compute the radial shift of the connecting spiral,

$$Y_1 = L_s \left(\frac{\phi_1}{3} - \frac{\phi_1^3}{42} + \frac{\phi_1^5}{1320} - \ldots\ldots \right)$$

$$Y_2 = L_s \left(\frac{\phi_2}{3} - \frac{\phi_2^3}{42} + \frac{\phi_2^5}{1320} - \ldots\ldots \right)$$

$$S_1 = Y_1 - R_1 (1 - \cos \phi_1)$$

$$S_2 = Y_2 - R_2 (1 - \cos \phi_2)$$

$$S = | S_2 - S_1 |$$

3- Compute the tangents lengths using eqs (2.33) and (2.34)

$$T_2 = \frac{(R_2-R_1-S)\cos\Delta_1 - R_2\cos\Delta}{\sin\Delta}$$

$$T_1 = R_2\sin\Delta - T_2\cos\Delta - (R_2-R_1-S)\sin\Delta_1$$

4- Compute the lengths of circular arcs,

$$L_1 = \pi R_1 (\Delta_1 - \phi_1) = \pi R_1 \beta_1/180$$

$$L_2 = \pi R_2 (\Delta_2 - \phi_2) = \pi R_2 \beta_2/180$$

5- Compute the coordinates of points PC, PT and C_1S (see Fig. 3-18) using eqs (3.5) and (3.6) :

a- Point PC

$$X_{PC} = X_{PI} + T_1 \sin(A_{z1} + 180)$$

$$Y_{PC} = Y_{PI} + T_2 \cos(A_{z1} + 180)$$

b- point C_1S

$$X_{C1S} = X_{PC} + d\sin(A_{z1} + \beta_1/2)$$

$$Y_{C1S} = Y_{PC} + d\cos(A_{z1} + \beta_1/2)$$

Where

$$\beta_1 = \Delta_1 - \phi_1$$

$$d = 2 R_1 \sin((\Delta_1-\phi_1)/2)$$

c- point PT

$$X_{PT} = X_{PI} + T_2 \sin(A_{z2})$$

$$Y_{PT} = Y_{PI} + T_2 \cos(A_{z2})$$

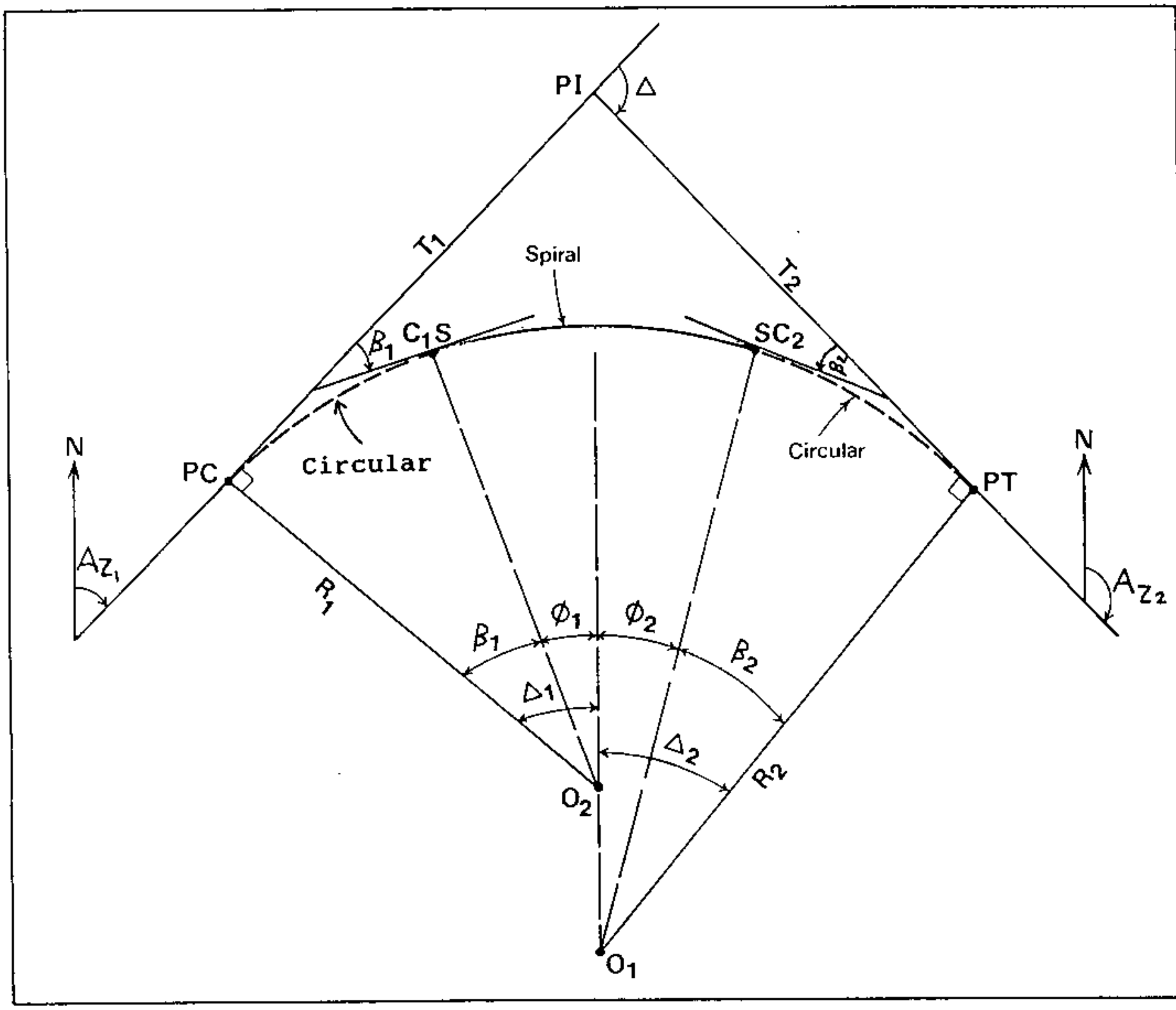


Figure (3-18): Showing How To Compute Coordinates Of The Main Points Of The 2-Centered Compound With One Spiral

6- Compute absolute coordinates for a group of points on the connecting spiral (Fig. 3-19):

a- Find the spiral central angle,

$$\phi_s = \phi_2 - \phi_1 = \frac{L_s}{60} * (D_2 - D_1)$$

b- For any point P on the spiral at a distance l from C_1S,

$$J = \frac{1}{2} * \frac{D_2 - D_1}{D_1} * \frac{1}{L}$$

Where

$$D_1 = \frac{30}{R_1} \text{ in radians}$$

$$D_2 = \frac{30}{R_2} \text{ in Radians}$$

The local coordinates x and y of point P are computed using equations (2.41) and (2.42) as shown below :

$$x_p = l - \frac{1}{2} \left(\frac{D_1}{30}\right)^2 l^3 \left(\frac{1}{3} + \frac{2}{4} K + \frac{1}{5} K^2\right)$$

$$+ \frac{1}{4!} \left(\frac{D_1}{30}\right)^4 l^5 * \left(\frac{1}{5} + \frac{4}{6} K + \frac{6}{7} K^2 + \frac{4}{8} K^3\right.$$

$$\left. + \frac{1}{9} K^4\right) - \cdots\cdots$$

$$y_p = \frac{D_1}{30} \; l^2 \left(\frac{1}{2} + \frac{1}{3} K\right) - \frac{1}{3!} \left(\frac{D_1}{30}\right)^3 l^4 \left(\frac{1}{4}\right.$$

$$\left. + \frac{3}{5} K + \frac{3}{6} K^2 + \frac{1}{7} K^3 \right) + \ldots\ldots\ldots$$

where $K = j\,l \quad (R_1 > R_2)$

$K = -j\,l \quad (R_1 < R_2)$

c- Find the length and azimuth of the line joining point C_1S to point P on the connecting spiral,

$$d = (x_p^2 + y_p^2)^{0.5}$$

$$\alpha = \tan^{-1} \frac{y_p}{x_p}$$

The Az.of the line C_1S—P is equal to (see Fig. 3-19),

$Az_1 + \beta_1 + \alpha$

d- Finally compute the absolute coordinates of point P using equations (3.5) and (3.6) :

$$X_P = X_{C1S} + d \sin (Az_1 + \alpha + \beta_1)$$

$$Y_P = Y_{C1S} + d \cos (Az_1 + \alpha + \beta_1)$$

7- Compute coordinates of point SC_2,

$$X_{SC2} = X_{PT} + d \sin (Az_2 + 180 - \alpha)$$

$$Y_{SC2} = Y_{PT} + d \cos (Az_2 + 180 - \alpha)$$

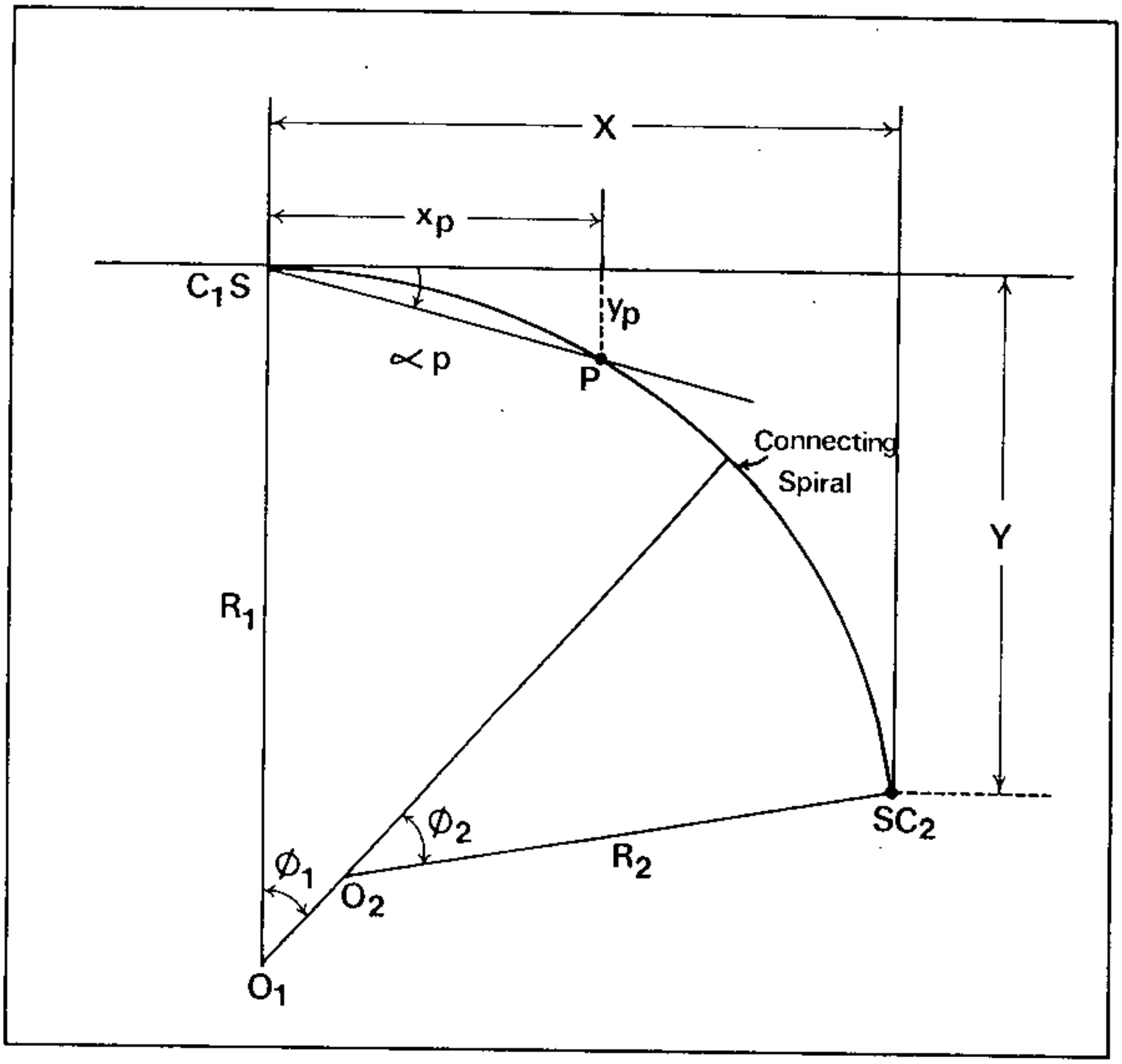


Figure (3-19): Showing How To Compute Coordinate For Points On The Combining Spiral Of The Two-Centered Compound Curve With One Spiral

Where

$$\alpha = (\Delta_2 - \phi_2)/2 = \beta_2/2$$

$$d = 2\ R_2 \sin((\Delta_2 - \phi_2)/2) = 2\ R_2 \sin \beta_2/2$$

8- Compute coordinates for a group of points on the left and right circular arcs in the same way described for simple circular curve.

9- After computing the absolute coordinates of all points on the curve, find the azimuth and distance of the line joining the PC to any point on the curve,

$$d = (\ \Delta X^2 + \Delta Y^2\ )^{0.5}$$

$$A_z = \tan^{-1} \frac{\Delta X}{\Delta Y}$$

3-7-2 Two-centered compound curve with 3 spirals

Given:

a- Absolute coordinates of PI (X_{PI}, Y_{PI}).

b- Azimuths of back and forward tangents at PI (A_{z1} & A_{z2})

c- Deflection angle Δ, at the PI.

d- Radii R_1 and R_2 or degrees of curve D_1 & D_2.

e- Δ_1 or Δ_2.

f- Length of entry spiral L_{s1} and exist spiral L_{s3}. If not given then the design speed and rate of change of radial acceleration will be given.

Procedure (see Fig. 2-17):

1- Find length of intermediate connecting spiral using equation (2.43),

$$L_{s2} = | L_{s3} - L_{s1} |$$

2- Find the central angle of each spiral,

Entry spiral , $\phi_1 = L_{s1} / 2 R_1$

Exit spiral , $\phi_2 = L_{s3}/2 R_2$

Intermediate spiral , $\phi_3 = L_{s2}/2 R_1$ and $\phi_4 = L_{s2}/2 R_2$

3- Compute the parameters, X,Y,X_o and S for the entry and exit spirals,using eqs (2.17),(2.18),(2.19) and (2.20).

$$X_1 = L_{s1} \left(1 - \frac{\phi_1^2}{10} + \frac{\phi_1^4}{216} \;\ldots\ldots\right)$$

$$Y_1 = L_{s1}\left(\frac{\phi_1}{3} - \frac{\phi_1^3}{42} + \frac{\phi_1^5}{1320} - \ldots\ldots\right)$$

$$X_2 = L_{s2} \left(1 - \frac{\phi_2^2}{10} + \frac{\phi_2^4}{216} \;\ldots\ldots\right)$$

$$Y_2 = L_{s2}\left(\frac{\phi_2}{3} - \frac{\phi_2^3}{42} + \frac{\phi_2^5}{1320} - \ldots\ldots\right)$$

$X_{o1} = X_1 - R_1 \sin \phi_1$

$X_{o2} = X_2 - R_1 \sin \phi_2$

$S_1 = Y_1 - R_2 (1-\cos \phi_1)$

$S_2 = Y_2 - R_2 (1-\cos \phi_2)$

2- Compute length of tangents using equation (2.44), (see Fig. 2-16):

Common tangent, $NJ = (R_1 + S_1) \tan \frac{\Delta_1}{2} + (R_2 + S_2) \tan \frac{\Delta_2}{2}$

Back tangent, $T_1 = NJ \frac{\sin \Delta_2}{\sin \Delta} + (R_1 + S_1) \tan \frac{\Delta_1}{2} + X_{o1}$

Forward tangent, $T_2 = NJ \frac{\sin \Delta_1}{\sin \Delta} + (R_2 + S_2) \tan \frac{\Delta_2}{2} + X_{o2}$

4- Find length of circular arcs,

Central angle of left circular arc, $\beta_1 = \Delta_1 - \phi_1 - \phi_3$

Central angle of right circular arc, $\beta_2 = \Delta_2 - \phi_2 - \phi_4$

Therefore

$L_1 = \pi R_1 \beta_1/180$

$L_2 = \pi R_2 \beta_2/180$

5- Compute the absolute coordinates of the main points TS_1, S_1C_1, C_1S_2, S_2C_2, C_2S_3 and S_3T using equations (3.5) and (3.6) as follows:

a- Point TS_1

$X_{TS1} = X_{PI} + T_1 \sin (A_{z1} + 180)$

$Y_{TS1} = Y_{PI} + T_2 \cos (A_{z1} + 180)$

b- Point S_1C_1

$$X_{S1C1} = X_{TS1} + d_1 \sin (A_{z1} + \alpha_1)$$

$$Y_{S1C1} = Y_{TS1} + d1 \cos (A_{z1} + \alpha_1)$$

Where

$$d_1 = (X_1^2 + Y_1^2)^{0.5}$$

$$\alpha_1 = \tan^{-1} \frac{Y_1}{X_1}$$

c- Point C_1S_2 (see Fig. 3-20)

$$X_{C1S2} = X_{S1C1} + d \sin (A_{z1} + \phi_1 + \beta_1/2)$$

$$Y_{C1S1} = Y_{S1C1} + d \sin (A_{z1} + \phi_1 + \beta_1/2)$$

where

$$d = 2 R_1 \sin \beta_1/2$$

d- Point S_2C_2

The local coordinates x and y of point S_2C_2 with respect to C_1S_2 as an origin and the tangent at C_1S_2 as an X-axis, are evaluated using the following two equations (eqs (2.45) and (2.46)):

$$x = (R_2 - R_1 - S) \sin \phi_3 + R_2 \sin (\phi_3 + \phi_4)$$

$$y = R_1 - (R_1 - R_2 - S) \cos \phi_3 - R_2 \cos (\phi_3 + \phi_4)$$

The length of chord between S_2C_2 and C_1S_2 is equal

$$d = (x^2 + y^2)^{0.5}$$

The deflection angle between the tangent at C_1S_2 and the above chord is found as follows :

$$\lambda = \tan^{-1} \frac{y}{x}$$

The azimuth of the tangent at C_1S_2 is $A_{z1} + \phi_1 + \beta_1$

The azimuth of the chord between C_1S_2 and S_2C_2 is equal to (see Fig. 3-20) $A_{z1} + \phi_1 + \beta_1 + \lambda$

Therefore the final absolute coordinates of point S_2C_2 are found as follows:

$$X_{S2C2} = X_{C1S2} + d \sin (A_{z1} + \phi_1 + \beta_1 + \lambda)$$

$$Y_{S2C2} = Y_{C1S2} + d \cos (A_{z1} + \phi_1 + \beta_1 + \lambda)$$

e- Point S_3T

$$X_{S3T} = X_{PI} + T_2 \sin (A_{z2})$$

$$Y_{S3T} = Y_{PI} + T_2 \cos (A_{z2})$$

f- Point C_2S_3

$$X_{C2S3} = X_{S3T} + d \sin (A_{z2} + 180 - \alpha)$$

$$Y_{C2S3} = Y_{S3T} + d \cos (A_{z2} + 180 - \alpha)$$

where

$$d = (X_2^2 + Y_2^2)^{0.5}$$

$$\alpha = \tan^{-1} \frac{Y_2}{X_2}$$

g- Point S_2C_2

$$\beta_2 = \Delta_2 - \phi_2 - \phi_4$$

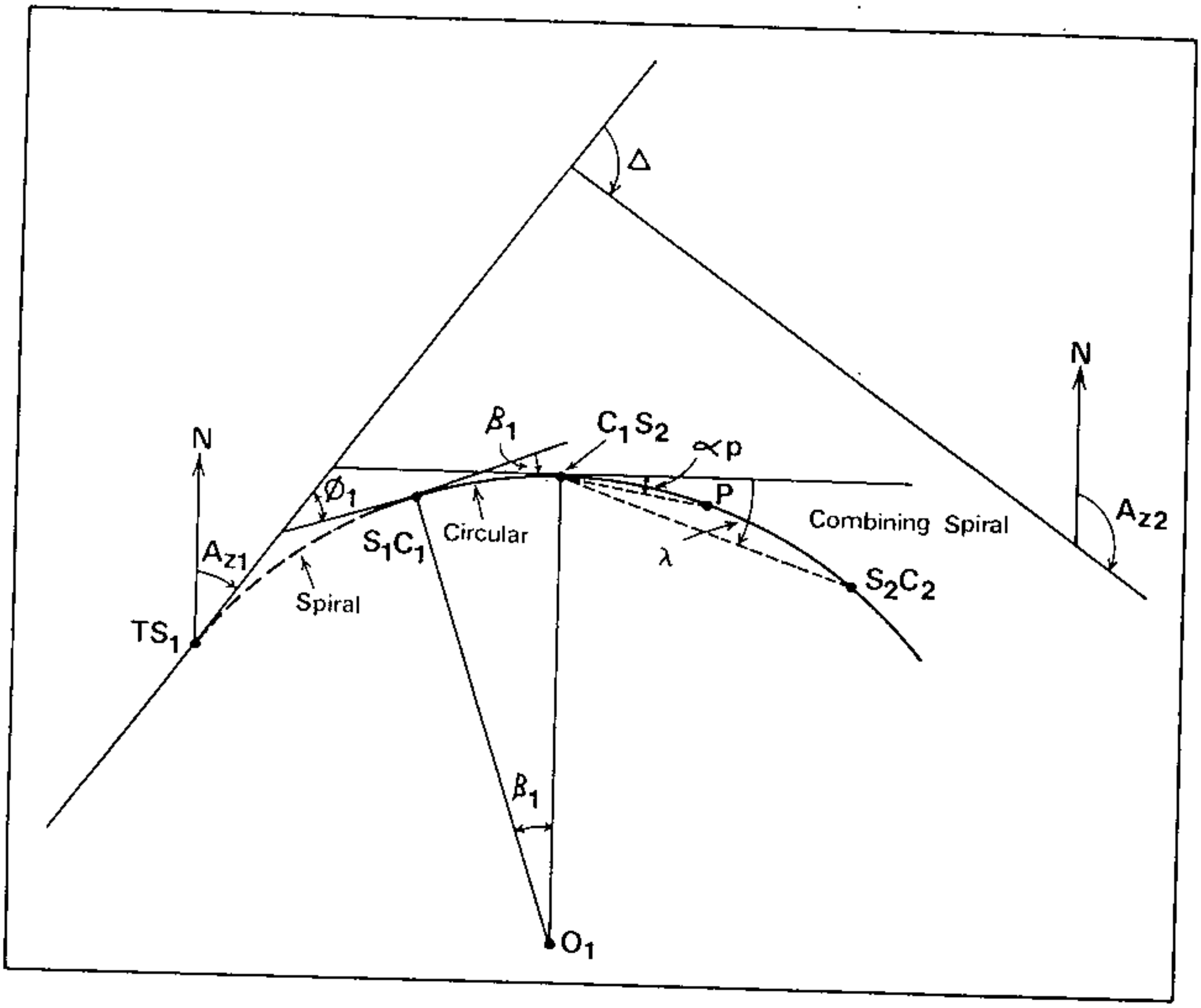


Figure (3-20): Showing How To Compute Coordinates For Points On The Combining Spiral Of The Two-Centered Compound Curve With 3 Spirals

$d = 2\ R_2 \sin \beta_2/2$

Therefore

$X_{S2C2} = X_{C2S3} + d \sin (Az_2 + 180 - \phi_2 - \beta_2/2)$

$Y_{S2C2} = Y_{C2S3} + d \cos (Az_2 + 180 - \phi_2 - \beta_2/2)$

6- Compute absolute coordinates for a group of points on each curve in a similar manner.

7- Compute the distance and azimuth of the line joining TS_1 to any other point on the curve.

3-8 REVERSE CURVES

3-8-1 Simple Reverse Curves

Given:-

a- Absolute coordinates of PI_1 (X_{PI1}, Y_{PI1}).

b- Absolute coordinates of PI_2 (X_{PI2}, Y_{PI2}).

c- Deflection angle Δ_1, at the PI_1.

d- Deflection angle Δ_2, at the PI_2.

e- Azimuth of back and forward tangents at PI (Az_1 & Az_2).

f- Azimuth of forward tangent at PI_2 (Az_3).

g- If R_1 is not equal to R_2, then R_1 or R_2 should be given.

Procedure (see Fig. 3-21):

1- Compute the unknown radius,

a- If $R_1 = R_2 = R$, then

$$T_1 = R \tan \Delta 1/2$$

$$T_2 = R \tan \Delta_2/2$$

The distance AB, between PI_1 & PI_2 is found using given coordinates,

$$AB = PI_1 - PI_2 = (\Delta X^2 + \Delta Y^2)^{0.5}$$

Therefore [1]

$$AB = T_1 + T_2 = R\left(\tan \frac{\Delta_1}{2} + \tan \frac{\Delta_2}{2}\right)$$

$$R = \frac{AB}{\tan \Delta_1/2 + \tan \Delta_2/2} \qquad (3.23)$$

b- If R_1 is not equal to R_2

$$T_1 = R_1 \tan \Delta_1/2$$

$$T_2 = AB - T_1$$

$$T_2 = AB - R_1 \tan \Delta_1/2$$

Therefore

$$R_2 = \frac{T_2}{\tan \Delta_2/2} = \frac{AB - R_1 \tan \Delta_1/2}{\tan \Delta_2/2} \qquad (3.24)$$

2- Find length of circular arcs,

$$L_{C1} = \pi R_1 \Delta_1/180$$

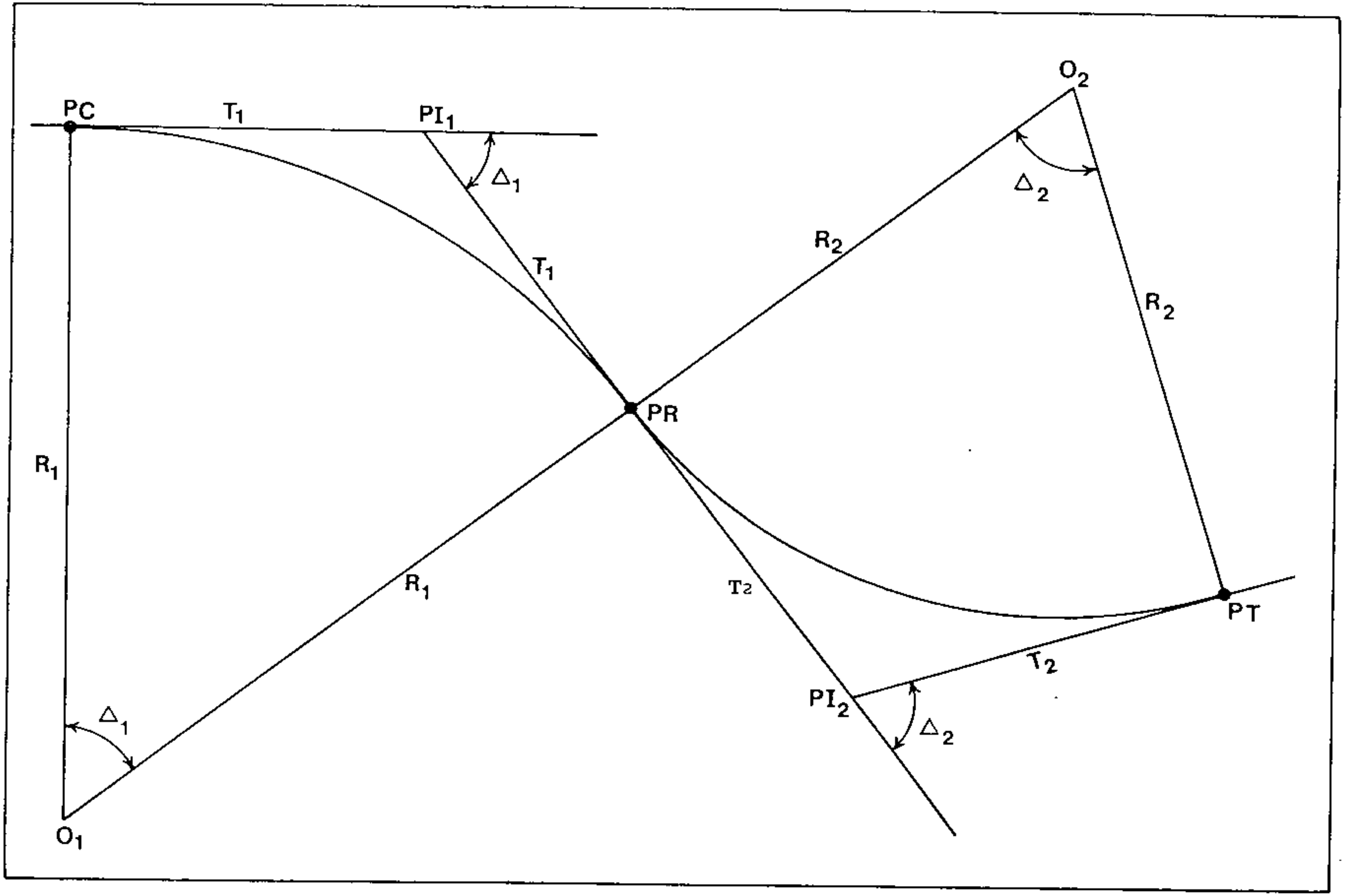


Figure (3-21): Simple Reverse Curve

$L_{C2} = \pi R_2 \Delta_2/180$

3- Find coordinates of main points using equations (3.5) and (3.6):

a- Point PC

$X_{PC} = X_{PI1} + T_1 \sin(A_{z1} + 180)$

$Y_{PC} = Y_{PI1} + T_1 \cos(A_{z1} + 180)$

b- Point PR

$X_{PR} = X_{PI1} + T_1 \sin(A_{z2})$

$X_{PR} = Y_{PI1} + T_1 \cos(A_{z2})$

c- Point PT,

$X_{PT} = X_{PI2} + T_2 \sin(A_{z3})$

$Y_{PT} = Y_{PI2} + T_2 \cos(A_{z3})$

4- Find coordinates for a group of points on each circular arc in the same manner described for simple circular curves.

5- Find the azimuth and distance of the line joining the PC to any other point on the reverse curve.

3-8-2 Spiraled Reverse Curves

Given:-

a- Absolutes coordinates of PI_1 and PI_2.

b- Azimuths of back and forward tangents at PI_1 (Az_1 & Az_2)

c-Azimuth of forward tangent at PI_2 (Az_3).

d- Deflection angle Δ_1, at the PI_1.

e- Deflection angle Δ_2, at the PI_2.

f- Radii R_1 and R_2.

g- Lengths of spirals L_{s1}, L_{s2} and L_{s3} if L_{s3} is not equal L_{s4} or length of spirals L_{s1} and L_{s2} only, if $L_{s3} = L_{s4}$

Procedure (see Figures 2-19 & 2-20):

1- Find the parameters of the first and second spirals using equations (2.11),(2.17),(2.18),(2.19) and (2.20) :

$$\phi_1 = \frac{L_{s1}}{2\ R_1}$$

$$\phi_2 = \frac{L_{s2}}{2\ R_1}$$

$$X_1 = L_{s1}\ (1 - \frac{\phi_1^{\ 2}}{10} + \frac{\phi_1^{\ 4}}{216}\ \ldots\ldots)$$

$$Y_1 = L_{s1}(\frac{\phi_1}{3} - \frac{\phi_1^{\ 3}}{42} + \frac{\phi_1^{\ 5}}{1320} - \ldots\ldots)$$

$$X_2 = L_{s2}\ (1 - \frac{\phi_2^{\ 2}}{10} + \frac{\phi_2^{\ 4}}{216}\ \ldots\ldots)$$

$$Y_2 = L_{s2}(\frac{\phi_2}{3} - \frac{\phi_2^{\ 3}}{42} + \frac{\phi_2^{\ 5}}{1320} - \ldots\ldots)$$

$$X_{o1} = X_1 - R_1 \sin \phi_1$$

$$X_{o2} = X_2 - R_1 \sin \phi_2$$

$$S_1 = Y_1 - R_1 \ (1-\cos \phi_1)$$

$$S_2 = Y_2 - R_2 \ (1-\cos \phi_2)$$

2- Compute the tangents lengths for the left part of the spiraled reverse curve using equations (2.22) and (2.23):

$$T_1 = X_{o1} + (R_1 + S_1) \tan \Delta_1/2 + \frac{S_2 - S_1}{\sin \Delta_1}$$

$$T_2 = X_{o2} + (R_1 + S_2) \tan \Delta_1/2 - \frac{S_2 - S_1}{\sin \Delta_1}$$

3- Find the distance between PI_1 and PI_2,

$$AB = PI_1 - PI_2 = (\Delta X^2 + \Delta Y^2)^{0.5}$$

where

$$\Delta X = | X_{PI1} - X_{PI2} |$$

$$\Delta Y = | Y_{PI1} - Y_{PI2} |$$

4- Compute length of fourth spiral L_{s4}

<u>Case 1</u> : L_{s3} is not equal L_{s4}

a- Compute parameters of third spiral,

$$\phi_3 = L_{s3}/2R_2$$

$$X_3 = L_{s3} \left(1 - \frac{\phi_3^{\ 2}}{10} + \frac{\phi_3^{\ 4}}{216} \ldots\ldots \right)$$

$$Y_3 = L_{s3} \left(\frac{\phi_3}{3} - \frac{\phi_3^{\ 3}}{42} + \frac{\phi_3^{\ 5}}{1320} \ldots \right)$$

$$X_{o3} = X_3 - R_2 \sin \phi_3$$

$$S_3 = Y_3 - R_2 (1-\cos \phi_3)$$

b- The length of backward tangent of the right part of the spiraled reverse curve is

$$T_3 = AB - T_2$$

$$T_3 = X_{o3} + (R_2 + S_3) \tan \Delta_2/2 + \frac{S_4 - S_3}{\sin \Delta_2}$$

Letting $H = \dfrac{S_4 - S_3}{\sin \Delta_2}$

Hence

$$T_3 = X_{o3} + (R_2 + S_3) \tan \Delta_2/2 + H$$

or

$$H = T_3 - X_{o3} - (R_2 + S_3) \tan \Delta_2/2$$

The only unknown in this equation is H, from which the shift of fourth spiral S_4, can be computed as follows:

$$S_4 = H \sin \Delta_2 + S_3$$

c- The length of the fourth spiral can now be computed using equation (2.48):

$$S = \frac{L_{s4}^{2}}{24 R} - \frac{L_{s4}^{4}}{2688 R^{3}}$$

This equation will be solved by trial and error

Case 2: $L_{s3} = L_{s4}$

For this case $S_3 = S_4$, thus we have an equal-tangent spiraled circular curve. The unknown in this case is the length of spiral.

$T_3 = AB - T_2$

The length of spiral can be computed by equation (2.50):

$$T_3 = \left(R + \frac{L_{s3}^{\;2}}{24\ R_2} - \frac{L_{s3}^{\;4}}{2688\ R_2^{\;3}} \right) \tan\frac{\Delta_2}{2} + \frac{L_{s3}}{2} - \frac{L_{s3}^{\;3}}{240\ R_2^{\;2}}$$

This equation will be solved by trial and error in order to get L_{s3} .

5- Compute lengths of circular arcs

$L_1 = \pi\ R_1\ (\ \Delta_1 - \phi_1 - \phi_2\ )/180$

$L_2 = \pi\ R_2\ (\ \Delta_2 - \phi_3 - \phi_4\ )/180$

6- Compute absolute coordinates of the main points TS_1, S_1C_1, C_1S_2, SS, S_3C_2, C_2S_4 and S_4T:

a- Point TS_1

$X_{TS1} = X_{PI1} + T_1 \sin\ (A_{z1} + 180)$

$Y_{TS1} = Y_{PI1} + T_1 \cos\ (A_{z1} + 180)$

b- Point S_1C_1

$X_{S1C1} = X_{TS1} + d \sin\ (A_{z1} + \alpha)$

$Y_{S1C1} = Y_{TS1} + d.\sin\ (A_{z1} + \alpha)$

Where

$$d = (X_1^2 + Y_1^2)^{0.5}$$

$$\alpha = \tan^{-1} Y_1/X_1$$

c- Point C_1S_2

$$X_{C1S2} = X_{S1C1} + d_1 \sin (Az_1 + \phi_1 + \lambda/2)$$

$$Y_{C1S2} = Y_{S1C1} + d_1 \cos (Az_1 + \phi_1 + \lambda/2)$$

Where

$$\lambda = \Delta_1 - \phi_1 - \phi_2$$

$$d_1 = 2 R_1 \sin \lambda/2$$

d- Point SS

$$X_{SS} = X_{PI1} + T_2 \sin (Az_2)$$

$$Y_{SS} = Y_{PI1} + T_2 \cos (Az_2)$$

e- Point S_3C_2

$$X_{S3C2} = X_{SS} + d \sin (Az_2 - \alpha)$$

$$Y_{S3C2} = Y_{SS} + d \cos (Az_2 - \alpha)$$

Where

$$d = (X_3^2 + Y_3^2)^{0.5}$$

$$\alpha = \tan^{-1} Y_3/X_3$$

f- Point C_2S_4

$$X_{C2S4} = X_{S3C2} + d_1 \sin (Az_2 - \phi_3 - \lambda/2)$$

$$Y_{C2S4} = Y_{S3C2} + d_1 \cos (Az_2 - \phi_3 - \lambda/2)$$

Where

$$\lambda = \Delta_2 - \phi_3 - \phi_4$$

$$d_1 = 2 R_2 \sin \lambda/2$$

g- Point $S_{4}T$

$$X_{S4T} = X_{PI2} + T_4 \sin (A_{z3})$$

$$Y_{S4T} = Y_{PI2} + T_4 \cos (A_{z3})$$

7- Compute the absolute coordinates for a selected group of points on each curve using the same manner employed in the design of simple circular and spiraled circular curves.

8- Compute the azimuth and the distance of the line that joins the point TS_1 to any other point on the spiraled reverse curve.

CHAPTER 4

APPLICATION (Solved Examples)

4-1 EXAMPLE ONE

4-1-1 Part One (Traverse adjustment)

Given: The connecting traverse shown in Fig. 4-1, is tied to the control points 0 & 1 at the start, and is tied also to the control points N & N+1 at the end. The absolute coordinates of the 4 control points are presented in Fig. 4-1. The length of all courses togather with the measured clockwise horizontal angles at the points of intersections(PIs) are given in the same figure.

Required:

1- Make a complete traverse adjustment.

2- Compute the adjusted coordinates of all the PIs.

3- Compute the final length and azimuth of each course.

4- Compute the deflection angle at each PI station.

Solution :

1- Compute the azimuth of control sides 0—1 and N—N+1.

n is the number of measured angles, in this case ,n = 8.

$$\alpha_{0-1} = \tan^{-1} \frac{X_1 - X_0}{Y_1 - Y_0} = \frac{86005.65 - 86233.67}{63521.79 - 63961.22}$$

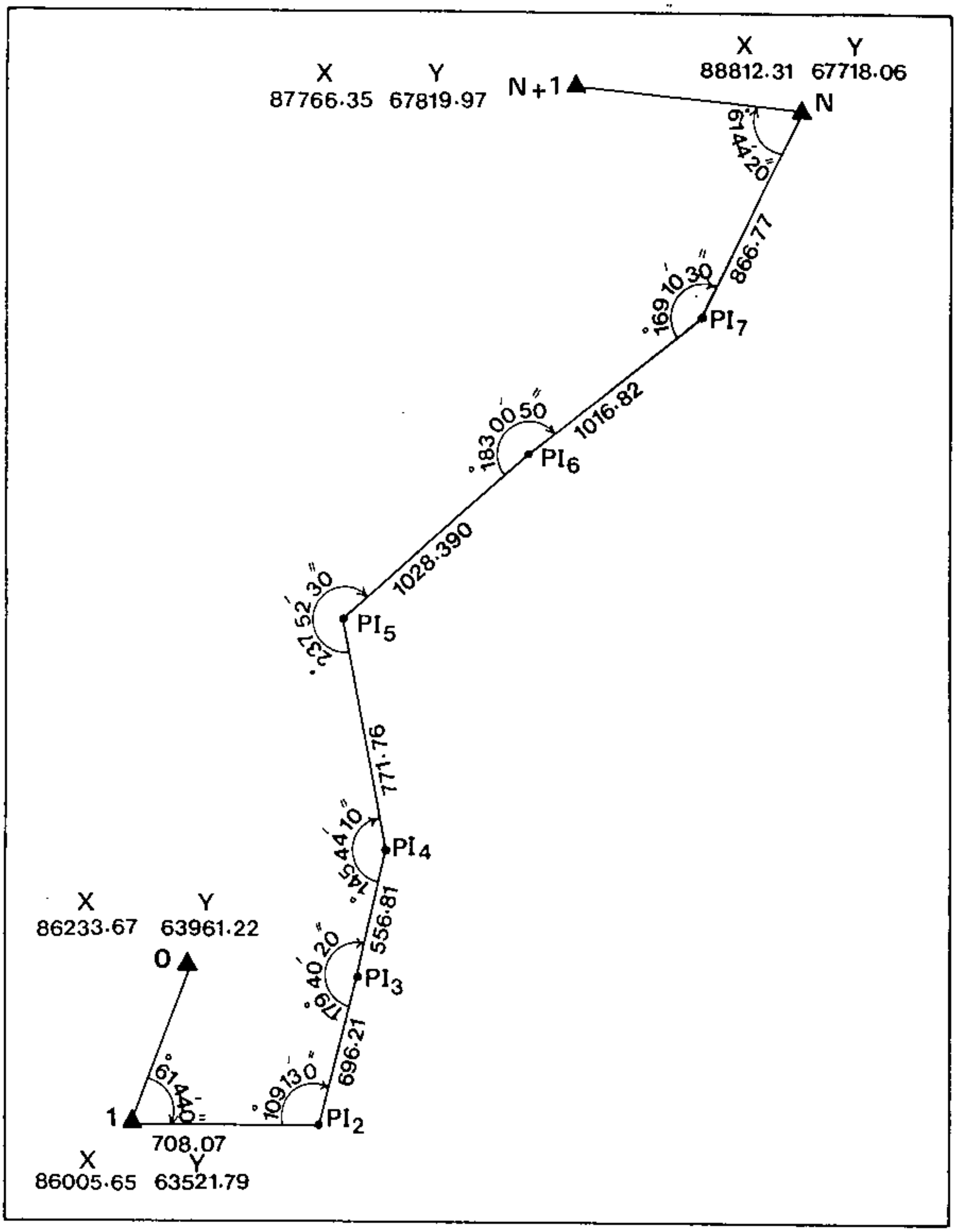


Figure (4-1): Connecting Traverse Of Example 1 (Part 1)

$= 207° 25' 29''$

$$\alpha_{N-N+1} = \tan^{-1} \frac{X_{N+1} - X_N}{Y_{N+1} - Y_N} = \frac{87766.35 - 88812.31}{67819.97 - 67718.06}$$

$= 275° 33' 54''$

2- Compute the unadjusted azimuth of all sides.

The following formula is used :

Az of side i = Az of side i–1+clockwise angle from side i–1 to side i $\pm$ 180

If the answer is greater than 360 degrees, then subtract 360 degrees from the total.

Az of side 1—2 = (207°25'29''+61°44'00''+ 180°)– 360°

= 89° 09' 29''

Az of side 2—3 = (89 09 29 + 109 13 00 + 180) – 360

= 18° 22' 29''

Az of side 3—4 = (18 22 29 + 179 40 20 + 180) – 360

= 18° 02' 49''

Az of side 4—5 = (18 02 49 + 145 44 10 + 180)

= 343° 46' 59''

Az of side 5—6 = (343 46 59 + 327 52 30 + 180) – 720

= 41° 39' 29''

Az of side 6—7 = (41 39 29 + 183 00 50 + 180) − 360

= 44° 40′ 19″

Az of side 7—N = (44 40 19 + 169 10 30 + 180) − 360

= 33° 50′ 49″

Az of side N—N+1 = (33 50 49 + 61 44 20 + 180)

= 275° 35′ 09″

3- Find the azimuth closure error.

The computed azimuth of side N—N+1 = 275° 35′ 09″

The known azimuth of side N—N+1 = 275° 33′ 54″

The closure error = ε = Computed Az. - Known Az.

ε = 275 35 09 − 275 33 54

= 00° 01′ 15″

4- Compute adjusted azimuths of all sides,

correction per angle = ε/N = (00° 01′ 15″)/8

= 9.4″

The correction that has to be applied to each side is found by using equation (3.4):

$$\text{correction} = -\left(\frac{i}{N}\right) * \varepsilon$$

The following table shows adjusted azimuth of all courses, Table (4-1): Adjusted Azimuths Of Example 1

side	Unadjusted Azimuth	correct.	Adjusted Azimuth
0—1	/	/	207° 25′ 29″
1—2	89° 09′ 29″	– 9	89 °09 ′20 ″
2—3	18 22 29	– 19	18 22 10
3—4	18 02 49	– 28	18 02 21
4—5	343 46 59	– 38	343 46 21
5—6	41 39 29	– 47	41 38 42
6—7	44 40 19	– 56	44 39 23
7—N(8	30 50 49	– 66	33 49 43
N—N+1	275 35 09	– 75	275 33 54

5- Compute the preliminary coordinates of all points ,

Here, we use the following two equations :

$$X_j = X_i + L_{ij} * \sin\ (\text{Az of side } _{ij})$$

$$Y_j = Y_i + L_{ij} * \cos\ (\text{Az of side } _{ij})$$

where L_{ij} is the length of side.

X(1) = 86005.65 (given)

Y(1) = 63521.79 (given)

X(2) = 86005.65 + 708.07*sin(89° 09′ 20″) = 86713.64

Y(2) = 63521.79 + 708.07*cos(89 09 20) = 63532.23

X(3) = 86713.64 + 696.21*sin (18 22 10) = 86933.05

Y(3) = 63532.23 + 696.21*cos (18 22 10) = 64192.96

X(4) = 86933.05 + 556.81*sin (18 02 21) = 87105.48

Y(4) = 64192.96 + 556.81*cos (18 02 21) = 64722.40

X(5) = 87105.48 + 771.76*sin (343 46 21) = 86889.81

Y(5) = 64722.40 + 771.76*cos (343 46 21) = 65463.41

X(6) = 86889.81 + 1028.39*sin (41 38 42) = 87573.19

Y(6) = 65463.41 + 1028.39*cos (41 38 42) = 66231.90

X(7) = 87573.19 + 1076.82*sin (44 39 23) = 88333.04

Y(7) = 66231.90 + 1076.83*cos (44 39 23) = 66997.88

X(N) = X(8) = 88333.04 + 866.77*sin (33 49 43)

= 88812.58

Y(N) = Y(8) = 66997.88 + 866.77*cos (33 49 43)

= 67717.91

6- Compute the closure error in the X and Y coordinates,

$\varepsilon x = X(N)_{\text{computed}} - X(N)_{\text{known}}$

$= X(8)_{\text{computed}} - X(8)_{\text{known}}$

$= 88812.58 - 88812.31 = 0.27$

$\varepsilon y = Y(8)_{\text{computed}} - Y(8)_{\text{known}}$

$= 67717.9 - 67718.06 = - 0.15$

Linear error $= (\varepsilon x^2 + \varepsilon y^2)^{0.5} = 0.31$

Relative linear error $= \dfrac{1}{\sum \text{length of side/linear error}}$

$$= \frac{1}{5704.83 \ / \ 0.31}$$

$$= \frac{1}{18000}$$

7- Compute the final adjusted coordinates,

correction to the X— Coord. of point i = $\frac{-l}{L} * \varepsilon x$

where

l : the cumulative length of courses up to point i.

L : the total length of all courses

Similarly

correction to the Y—Coord. of point i = $\frac{-l}{L} * \varepsilon y$

Therefore the following table gives the final adjusted X--coordinates of all points :

Table (4-2): Adjusted X-coordinates Of Example 1

point No	cummulative distance	preliminary X--Coord	correct. to X	Adjusted X--Coord
1	0	86005.65	0	86005.65
2	708.07	86713.64	-0.03	86713.61
3	1404.28	86933.05	-0.07	86932.98
4	1961.09	87105.48	-0.09	87105.39
5	2732.85	86889.81	-0.13	86889.68
6	3761.24	87573.19	-0.18	87573.01
7	4838.06	88330.04	-0.23	88329.81
8	5704.83	88812.58	-0.27	88812.31

Similarly the final adjusted Y-Coordinates are obtained from the following table:

Table (4-3): Adjusted Y-coordinates Of Example 1

point no	preliminary Y--Coord.	correction to Y	Adjusted Y--cood
1	63521.79	0	63521.79
2	63532.23	0.02	63532.25
3	64192.96	0.04	64193.00
4	64722.40	0.05	64722.45
5	65463.41	0.07	65463.48
6	66231.90	0.10	66232.00
7	66997.88	0.13	66998.01
8	67717.91	0.15	67718.06

8- Compute the final azimuth and length of all courses, based on the adjusted coordinates, the following two formulae are used :

$$\text{Az of side } ij = \tan^{-1} \frac{X_j - X_i}{Y_j - Y_i}$$

$$d_{ij} = ((X_j - X_i)^2 + (Y_j - Y_i)^2)^{0.5}$$

For example the length of side 3—4 is equal to

$$d_{34} = ((87105.39-86932.98)^2+(64722.45 - 64193.00)^2)^{0.5}$$

$$= 556.815$$

Similarly

$$\text{Az of side 3—4} = \tan^{-1} \frac{87105.39 - 86932.98}{64722.45 - 64193.00}$$

$$= 18° \; 02' \; 14''$$

The following table shows the final azimuth and length of all courses.

Table (4-4): Final Azimuths And Lengths Of Sides Of Example 1

Side	Final Length	Final Azimuth
1—2	708.037	89 °09 ´13 ´´
2—3	696.214	18 21 58.5
3—4	556.815	18 02 14
4—5	771.788	343 46 12
5—6	1028.379	41 38 31
6—7	1076.809	44 39 12.5
7—N(8)	866.763	33 49 32

9- Compute the deflection angles at the points of intersections (PIs) of the traverse,

Deflection angle at point i equals the absolute difference in azimuth beween the backward and forward sides. If the difference is > 180 then the answer must be subtracted from 360.

The deflection angle at point 2 = 89°09´20´´ – 18°22´10´´

= 70° 47´ 10´´

The deflection angle at point 3 = 18 22 10 – 18 02 21

= 00 19 49

The deflection angle at point 4 = 343 46 21 – 18 02 21

= 325 44 00

since the answer is more than 180,

The deflection angle at point 4 = 360 00 00 – 325 44 00

= 34 16 00

The deflection angle at point 5 = 343 46 21 – 41 38 42

= 302 07 39 (>180)

= 57 52 21

The deflection angle at point 6 = 44 39 23 – 41 38 42

= 03 00 45

The deflection angle at point 7 = 44 39 23 – 33 49 43

= 10 49 40

4-1-2 Part Two (Design Of Equal-Tangent Spiraled Circular Curve)

Given the connecting traverse of part 1 whose final adjusted coordinates have been computed, it is required to design an equal-tangent spiraled circular curve at point 4 (see Fig. 4-2).

Given Data :

X--Coord. of PI_4 = 87105.39

Y--Coord. of PI_4 = 64722.45

Azimuth of backward tangent at PI_4 = Az_1 = 18° 02′ 14″

Azimuth of forward tangent at PI_4 = Az_2 = 343 46 12

Deflection angle at PI_4 = Δ = 34 16 00

Length of spiral curve = 150 m

Radius of circular curve = 600 m

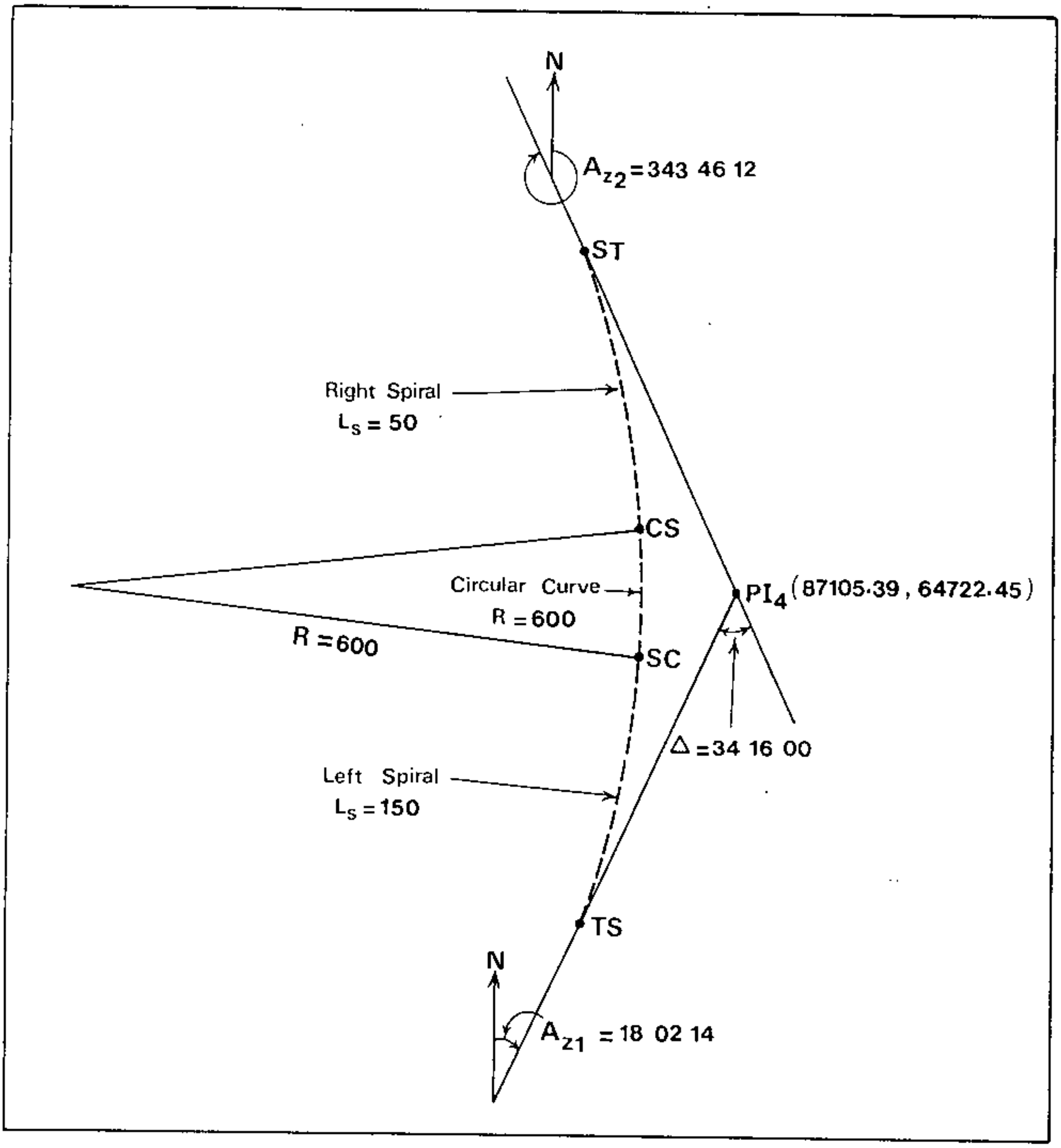


Figure (4-2): Equal-Tangent Spiraled Circular Curve Of Example 1 (Part 2)

Solution (See Fig. 2-7)

1- Compute the tangent length,

spiral central angle $\phi = \frac{L_s}{2\ R} = \frac{150}{2*600} = 0.125$ radian

$= 07° \ 09' \ 43''$

The local coordinates of point SC with respect to TS as an origin and the back tangent as an X—axis, are found as follows

$$X = L_s \ (1 - \frac{\phi^2}{10} + \frac{\phi^4}{216} - \ldots\ldots)$$

$$X = 150 \ (1 - \frac{0.125^2}{10} + \frac{0.125^4}{216} \ )$$

$= 149.766$ m

$$Y = L_s \ (\frac{\phi}{3} + \frac{\phi^3}{42} + \frac{\phi^5}{1320} - \ldots\ldots\ldots)$$

$$Y = 150 \ (\frac{0.125}{3} - \frac{0.125^3}{42} + \frac{0.125^5}{1320} \ )$$

$= 6.243$ m

The distance Xo and shift S will be now computed,

$X_o = X - R \sin \phi$

$= 149.765 - 600* \sin (07° \ 09' \ 43'')$

$= 74.960$ m

$S = Y - R \ (1 - \cos \phi \ )$

$= 6.243 - 600 * (1 - \cos 07\ \ 09\ \ 43\ )$

$= 1.562$ m

Therefore the length of tangent is computed as follows:

$$\text{Tangent length} = T = X_O + (R+S)\ \tan \frac{\Delta}{2}$$

$$= 74.960 + (600 + 1.562)\ \tan \frac{34\ \ 16\ \ 00}{2}$$

$$= 260.408 \text{ m}$$

2- Compute the absolute coordinates of point TS

$$X_{TS} = X_{PI} + T \sin\ (Az_1 + 180)$$

$$= 87105.39 + 260.408 * \sin\ (18°\ 02'\ 14'' + 180°)$$

$$= 87024.759 \text{ m}$$

$$Y_{TS} = Y_{PI} + T \cos\ (Az_1 + 180)$$

$$= 64722.45 + 260.408 * \cos\ (18\ \ 02\ \ 14\ \ + 180\ )$$

$$= 64474.84 \text{ m}$$

3- Compute the absolute coordinates for a group of points on the left spiral curve.

a- Compute the partial spiral angle θ , for each arc length of the spiral,

let $l = R/40 = 600/40 = 15$ m

Thus

$l_1 = 15$, $l_2 = 30$, $l_3 = 45$, $l_4 = 60$ m , . . .

$$\theta = (\frac{l}{L_s})^2 * \phi$$

Therefore θ for each point on the spiral is computed as follows :

$$\theta_1 = (15/150)^2 * 0.125 = 0.00125 \text{ radian}$$

$$\theta_2 = (30/150)^2 * 0.125 = 0.005 \text{ radian}$$

$$\theta_3 = (45/150)^2 * 0.125 = 0.01125$$

similarly

$\theta_4 = 0.020$

$\theta_5 = 0.03125$

$\theta_6 = 0.045$

$\theta_7 = 0.06125$

$\theta_8 = 0.080$

$\theta_9 = 0.10125$

$\theta_{10} = 0.125$

b- Compute the local coordinates of the points on the left spiral, assuming that point TS is the origin and the tangent PI—TS is the X--axis (see Fig. 4-3).

$$x_1 = l_1 * \left(1 - \frac{\theta_1^2}{10} + \frac{\theta_1^4}{216} \right)$$

$$y_1 = l_1 * \left(\frac{\theta_1}{3} - \frac{\theta_1^3}{42} + \frac{\theta_1^5}{1320} \right)$$

hence

$$x_1 = 15 * \left(1 - \frac{0.00125^2}{10} + \frac{0.00125^4}{216}\right) = 14.99 \text{ m}$$

$$y_1 = 15*\left(\frac{0.00125}{3} - \frac{0.00125^3}{42} + \frac{0.00125^5}{1320}\right) = 0.006 \text{ m}$$

$$x_2 = l_2 * \left(1 - \frac{\theta_2^2}{10} + \frac{\theta_2^4}{216}\right)$$

$$y_2 = l_2 * \left(\frac{\theta_2}{3} - \frac{\theta_2^3}{42} + \frac{\theta_2^5}{1320}\right)$$

hence

$$x_2 = 30 * \left(1 - \frac{0.005^2}{10} + \frac{0.005^4}{216}\right) = 29.999 \text{ m}$$

$$y_2 = 30 * \left(\frac{0.005}{3} - \frac{0.005^3}{42} + \frac{0.005^5}{1320}\right) = 0.05 \text{ m}$$

$$x_3 = 45 * \left(1 - \frac{0.01125^2}{10} + \frac{0.01125^4}{216}\right) = 44.999 \text{ m}$$

$$y_3 = 45 * \left(\frac{0.01125}{3} - \frac{0.01125^3}{42} + \frac{0.01125^5}{1320}\right) = 0.169 \text{ m}$$

In a similar way

$x_4 = 59.998$ m , $y_4 = 0.399$ m

$x_5 = 74.993$, $y_5 = 0.781$

$x_6 = 89.982$, $y_6 = 1.35$

$x_7 = 104.961$, $y_7 = 2.144$

$x_8 = 119.923$, $y_8 = 3.198$

$x_9 = 134.858$, $y_9 = 4.61$

$x_{10} = 149.766$, $y_{10} = 6.243$

c- Compute deflction angle for each point on left spiral as measured from the back tangent PI-Ts (Fig. 4-3),

$$\alpha_1 = \tan^{-1}\frac{y_1}{x_1} = \tan^{-1}\frac{0.006}{14.999} = 00° \; 01' \; 22.5''$$

$$\alpha_2 = \tan^{-1}\frac{y_2}{x_2} = \tan^{-1}\frac{0.050}{29.999} = 00 \quad 05 \quad 44$$

$$\alpha_3 = \tan^{-1}\frac{y_3}{x_3} = \tan^{-1}\frac{0.169}{44.999} = 00 \quad 12 \quad 55$$

Likewise,

$\alpha_4 = 00° \; 22' \; 52''$, $\alpha_5 = 00° \; 35' \; 48''$

$\alpha_6 = 00 \quad 51 \quad 34$, $\alpha_7 = 01 \quad 10 \quad 13$

$\alpha_8 = 01 \quad 31 \quad 39$, $\alpha_9 = 01 \quad 57 \quad 28$

$\alpha_{10} = 02 \quad 23 \quad 13$

Note that α_{10} is roughly equals $\frac{\phi}{3} = \frac{07 \quad 09 \quad 43}{3}$

$$= 02° \; 23' \; 13''$$

d- Compute the length of chords that connect point TS to any other point on the left spiral (see Fig. 4-3),

$$s_1 = (x_1^2 + y_1^2)^{0.5} = (14\ 999^2 + 0.006^2)^{0.5} = 14.999 \text{ m}$$

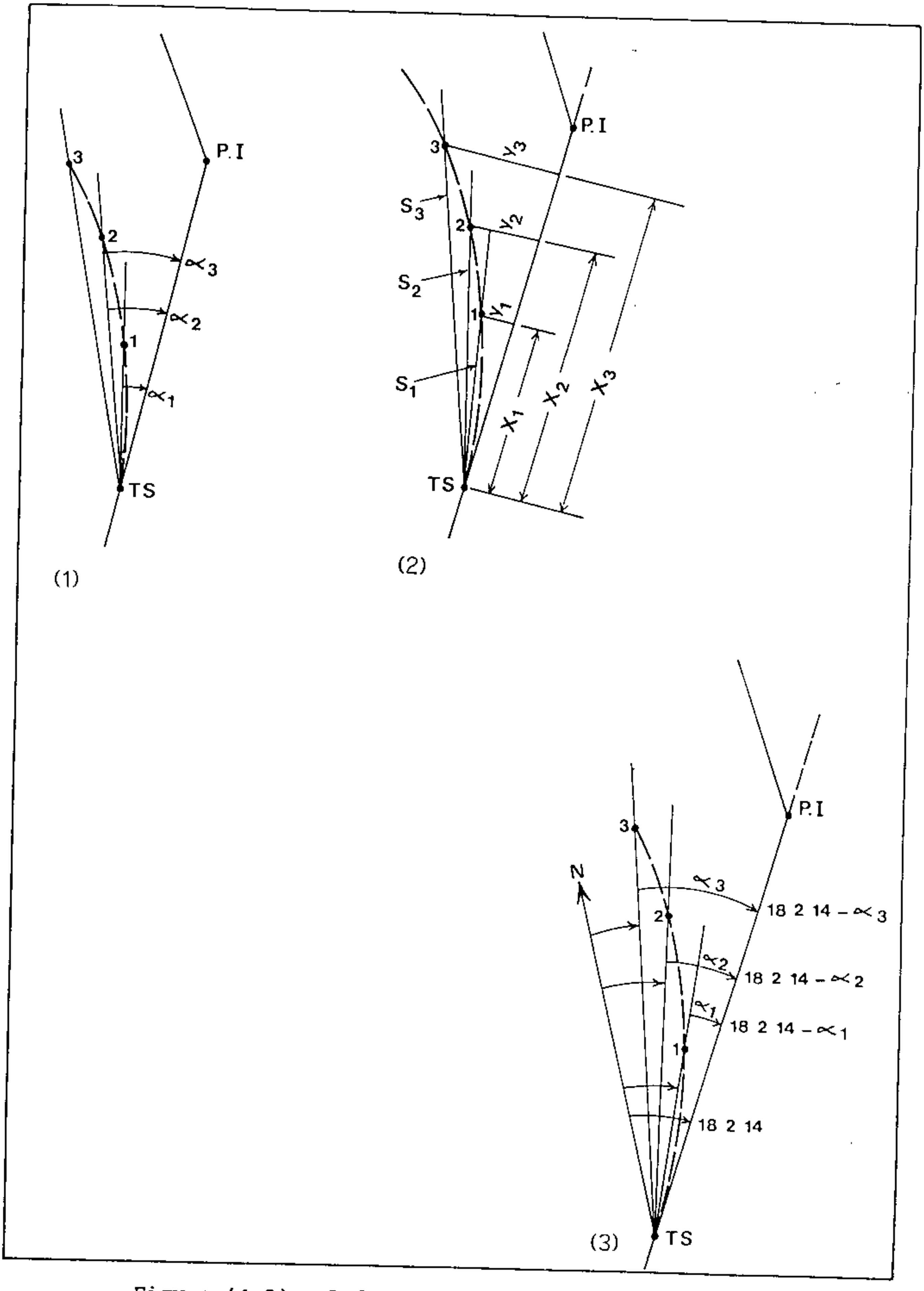


Figure (4-3): Left Spiral Of Example 1 (Part 2)

$s_2 = (x_2^2 + y_2^2)^{0.5} = (29.999^2 + 0.05^2)^{0.5} = 29.999$ m

$s_3 = (x_3^2 + y_3^2)^{0.5} = (44.999^2 + 0.169^2)^{0.5} = 44.999$ m

similarly

$s_4 = 59.999$, $s_5 = 74.997$

$s_6 = 89.992$, $s_7 = 104.983$

$s_8 = 119.966$, $s_9 = 134.896$

$s_{10} = 134.936$

Note that s_{10} is the distance between points TS & SC

e- Compute the azimuth of all the chords that connect point TS to any point on the spiral (see Fig. 4-3),

Az of chord TS--PI = $Az_1 = 18° \ 02' \ 14''$

Az of chord TS--1 $= Az_1 - \alpha_1$

$= 18 \ 02 \ 14 - 00 \ 01 \ 22.5 = 18° \ 00' \ 51''$

Az of chord TS--2 $= Az_1 - \alpha_2$

$= 18 \ 02 \ 14 - 00 \ 05 \ 44 = 17 \ 56 \ 30$

Az of chord TS--3 $= Az_1 - \alpha_3$

= 18 02 14 − 00 12 55 = 17 49 19

Az of chord TS--4 = Az1 − α_4

= 18 02 14 − 00 22 52 = 17 49 19

similarly

Az of chord TS--6 = 17° 10´ 40´´

Az of chord TS--7 = 16 52 01

Az of chord TS--8 = 16 30 35

Az of chord Ts--9 = 16 04 46

Az of chord TS--10 = 15 39 01

Note that the azimuth of chord TS-10 is equal to the azimuth of the line joining TS and SC.

f- Compute the absolute coordinates for a group of points on the left spiral,

$X_1 = X_{TS} + s_1 * \sin$ (Az of TS--1)

$Y_1 = Y_{TS} + s_1 * \cos$ (Az of TS--1)

X_1 = 87024.759 + 14.999* sin(18°00´51.5´´)= 87029.398

Y_1 = 64474.840 + 14.999 *cos(18 00 51.5) = 64489.103

X_2 = 87024.759 + 29.999 * sin (17 56 30)= 87034.000

Y_2 = 64474.84 + 29.999 * cos (17 56 30)= 64503.380

X_3 = 87024.759 + 44.999 * sin (17 49 19)= 87038.531

Y_3 = 64474.84 + 44.999 * cos (17 49 19)= 64517.680

X_4 = 87024.759 + 59.999 * sin (17 39 22)= 87042.957

Y_4 = 64474.84 + 59.999 * cos (17 39 22)= 64532.012

X_5 = 87024.759 + 74.997 * sin (17 26 26)= 87047.237

Y_5 = 64474.84 + 74.997 * cos (17 26 26)= 64546.390

X_6 = 87024.759 + 89.999 * sin (17 10 40) = 87051.337

Y_6 = 64474.84 + 89.999 * cos (17 10 40) = 64560.818

X_7 = 87024.759 + 104.983 * sin (16 52 01) = 87055.220

Y_7 = 64474.840 + 104.983 * cos (16 52 01) = 64575.307

X_8 = 87024.759 + 119.966 * sin (16 30 35) = 87058.851

Y_8 = 64474.840 + 119.966 * cos (16 30 35) = 64589.860

X_9 = 87024.759 + 134.936 * sin (16 04 46) = 87062.132

Y_9 = 64474.840 + 134.936 * cos (16 04 46) = 64604.492

X_{10} = X_{SC} = X_{TS} + s_{10} * sin (Az of TS--10)

Y_{10} = Y_{SC} = Y_{TS} + s_{10} * cos (Az of TS--10)

X_{10}=X_{SC}=87024.759+149.896 * sin(15 39 01) = 87065.196

Y_{10}=Y_{SC}=64474.840+149.896 * cos(15 39 01) = 64619.179

Therefore the following table shows the final absolute coordinates for points on left spiral.

Table (4-5): Final Coordinates For Points On Left Spiral Of Example 1

Point No.	X--coordinate	Y-coordinate
TS	87024.759	64474.840
1	87029.398	64490.790
2	87034.000	64503.380
3	87038.531	64517.680
4	87042.957	64532.012
5	87047.237	64546.390
6	87051.337	64560.818
7	87055.220	64575.307
8	87058.851	64589.860
9	87062.132	64604.492
10(SC)	87065.196	64619.179

4- Compute the coordinates for a group of points on the circular curve,

a- Determine the central angle β, of the circular arc,

$$\beta = \Delta - 2\ \phi$$

$$= 34\quad 16\quad 00 - 2 * 07\quad 09\quad 43$$

$$= 19^{\circ}\ 56'\ 34''$$

b- Compute the length of circular curve

$$L = \pi * R * \beta/180$$

$$= \pi * 600 * (19\quad 56\quad 34)/180$$

$$= 208.840 \text{ m}$$

c- Compute the deflection angles for a group of points on the circular curve. The deflection angle is measured from the tangent at point SC (see Fig. 4-4). The circular arc is divided into equal parts so that

$$l = R/20 = 600/20 = 30 \text{ m}$$

Thus we have

$l_1 = 30$, $l_2 = 60$, $l_3 = 90$, $l_4 = 120$, $l_5 = 150$,

$l_6 = 180$, $l_7 = 208.84 - 6 * 30 = 28.84$ m

The deflection angle is computed using the following equation :

$$\alpha = \frac{180}{2\pi} * \frac{l}{R}$$

Therefore we have

$$\alpha_1 = \frac{180}{2\pi} * \frac{l_1}{R} = \frac{180}{2\pi} * \frac{30}{600} = 01° \; 25' \; 57''$$

$$\alpha_2 = \frac{180}{2\pi} * \frac{l_2}{R} = \frac{180}{2\pi} * \frac{60}{600} = 02 \quad 51 \quad 53$$

similarly

$\alpha_3 = 04 \quad 17 \quad 50$, $\alpha_4 = 05 \quad 43 \quad 46.5$

$\alpha_5 = 07 \quad 09 \quad 43$, $\alpha_6 = 08 \quad 35 \quad 40$

$$\alpha_7 = \alpha_{SC} = \frac{180}{2\pi} * \frac{208.84}{600} = 09 \quad 58 \quad 17$$

Notice that $\alpha_7 = \alpha_{sc} = \beta/2$

d- Compute the length of chords that connect point SC to any other point on the circular arc (Fig. 4-4), The chord length d is computed by this equation

$$d = 2 * R * \sin \alpha$$

thus we have

$d_1 = 2 * R * \sin \alpha_1 = 2 * 600 * \sin (01 \quad 25 \quad 57 \quad)$

$= 29.999$ m

$d_2 = 2 * R * \sin \alpha_2 = 2 * 600 * \sin (02 \quad 51 \quad 53 \;)$

$= 59.974$

$d_3 = 2 * R * \sin \alpha_3 = 2 * 600 * \sin (04 \quad 17 \quad 50 \;)$

$= 89.916$

Similarly

$d_4 = 119.800$, $d_5 = 149.609$

$d_6 = 179.327$, $d_7 = 207.788$

e- Compute the azimuth of all chords that connect point SC to any point on the circular arc (see Fig 4-4),

Az.of tangent at SC = Az of back tangent(Az_1) $- \phi$

$= 18° \; 02' \; 14'' - 07° \; 09' \; 43''$

$= 10 \quad 52 \quad 31$

thus we have

Az of chord SC--1 = Az of tangent at SC − α_1

= 10 52 31 − 07 09 43

= 09° 26´ 34´´

Az of chord SC--2 = Az of tangent at SC − α_2

= 10 52 31 − 02 51 53

= 08 00 38

Similarly

Az of chord SC--3 = 10 52 31 − 04 17 50

= 06 34 41

Az of chord SC--4 = 10 52 31 − 05 43 45.5

= 05 08 44.5

Az of chord SC--5 = 03 42 48

Az of chord SC--6 = 02 16 51

Az of chord SC--7 = Az Of chord SC--CS ≐00 54 14

f- Compute absolute coordinates for a group of points on the circular curve,

Remember that X_{SC} = 87065.196 and Y_{SC} = 64619.179

$X_1 = X_{SC} + d_1 * \sin$ (Az of chord SC--1)

= 87065.196 + 29.999 sin (09° 26´ 34´´)= 87070.118

$Y_1 = Y_{SC} + d_2 * \cos$ (Az of chord SC--1)

= 64619.179 + 29.999 cos (09 26 34)= 64648.772

$X_2 = X_{SC} + d_2 * \sin$ (Az of chord SC--2)

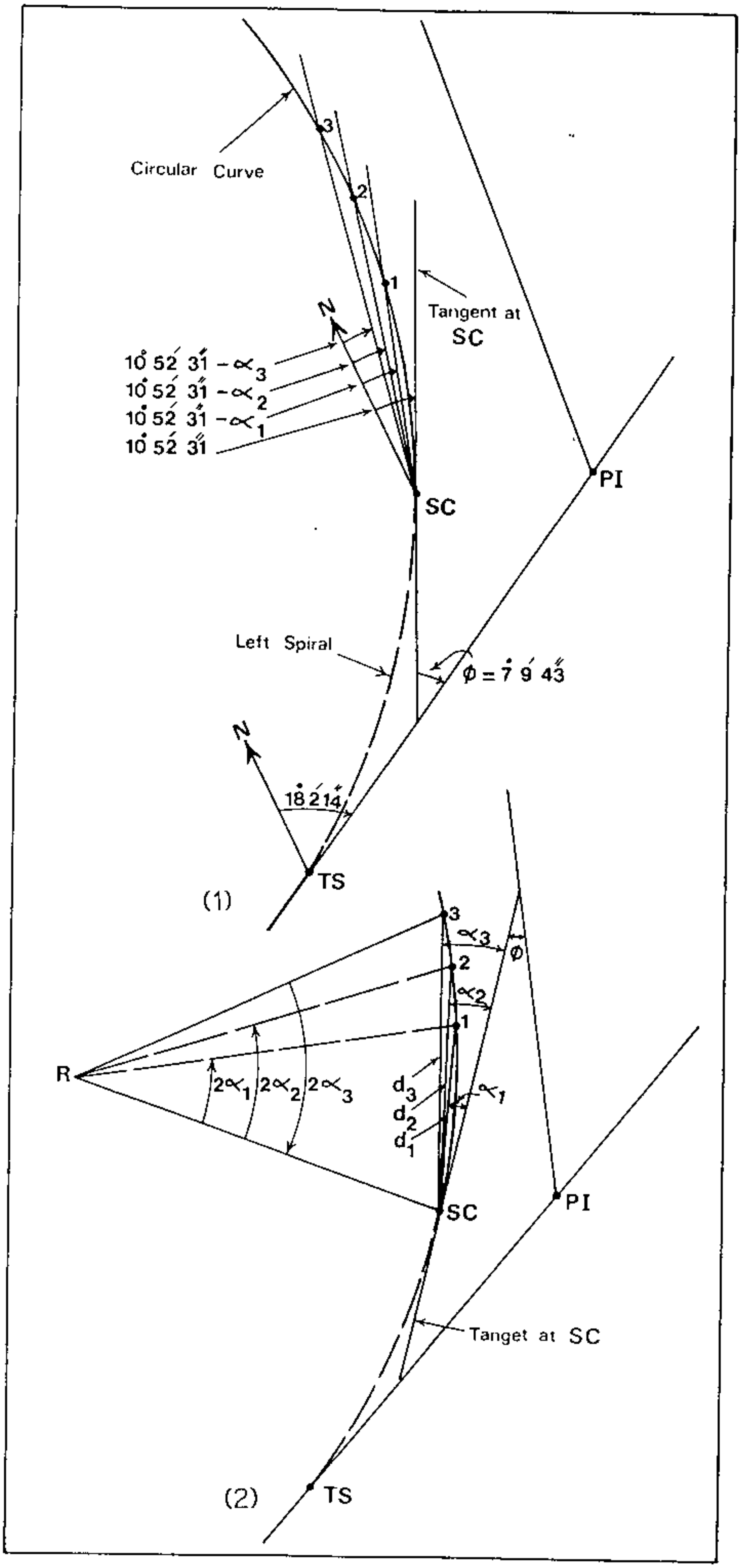


Figure (4-4): Circular Curve Of Example 1 (Part 2)

= 87065.196 + 59.974 sin (08 00 38)= 87073.554

Y_2 = Y_{SC} + d_2 * cos (Az of chord SC--3)

= 64619.179 + 59.974 * cos (08 00 38)= 64678.568

Similarly

X_3 = 87065.196 + 89.916 * sin (06 34 41)= 87075.497

Y_3 = 64619.179 + 89.916 * cos (06 34 41)= 64708.503

X_4 = 87065.196 + 119.80 * sin (05 08 44.5)=87075.94

Y_4 = 64619.179 + 119.80 * cos(05 08 44.5)=64738.496

X_5 = 87065.196 + 149.609 * sin (03 42 48)=87074.885

Y_5 = 64619.179 + 149.609 * cos (03 42 48)=64768.474

X_6 = 87065.196 + 179.327 * sin (02 16 51)=87072.333

Y_6 = 64619.179 + 179.327 * cos (02 16 51)=64798.364

X_7 = X_{SC} = 87065.196 +207.788 sin(00 54 14)=87068.474

Y_7 = X_{SC} = 64619.179 +207.788 cos(00 54 14)=64826.941

The following table shows the final absolute coordinates for points on the circular curve :

Table (4-6): Final Coordinates For Points On Circular Arc Of Example 1

Point No	X--Coordinate	Y--Coordinate
SC	87065.196	64619.179
1	87070.118	64648.772
2	87073.554	64678.568
3	87075.497	64708.503
4	87075.940	64738.496
5	87074.885	64768.474
6	87072.333	64798.364
7(CS)	87068.474	64826.941

5- Compute absolute coordinates of points on right spiral.

a- Compute absolute coordinates of point ST

Given Az.of forward tangent (PI-ST)= $A_{z2}=343°46'12''$

Azimuth of line ST-PI =(180 + 343 46 12) – 360

= 163 46 12

X_{ST} = X_{PI} + T sin (Az of line PI--ST)

Y_{ST} = Y_{PI} + T cos (Az of line PI--ST)

X_{ST} = 87105.39 + 260.408 sin (343 46 12) = 87032.608

Y_{ST} = 64722.45 + 260.408 cos(343 46 12)= 64972.480

b- Since the left and right spirals are of equal length, then the local coordinates for points on the right spiral relative to point ST are the same as for left spiral. Consequently, the deflection angles and chords lengths for the points on right spiral are identical to those on left spiral. This means that we can use the same values $\alpha_1, \alpha_2, \alpha_3, \ldots \alpha_{10}$ and s_1, s_2, $s_3, \ldots s_{10}$, as computed for the left spiral.

c- Compute the azimuth for the line joining ST to all points on right spiral (see Fig 4-5),

Az of chord ST--1 = Az of ST--PI + α_1

=163 46 12+00 01 22.5 =163 47 34.5

Az of chord ST--2 = Az of ST--PI + α_2

= 163 46 12 + 00 05 44 = 163 51 56

Az of chord ST--3 = Az of ST--PI + α_3

= 163 46 12 + 00 12 55 = 163 69 07

Similarly

Az of chord ST--4 = 164 09 04

Az of chord ST--5 = 164 22 00

Az of chord ST--6 = 164 37 46

Az of chord ST--7 = 164 56 25

Az of chord ST--8 = 165 17 51

Az of chord ST--9 = 165 43 40

Az of chord ST-10 = Az of ST-CS = Az of ST-PI + α_{10}

= 163 46 12 + 02 23 13

= 166 09 25

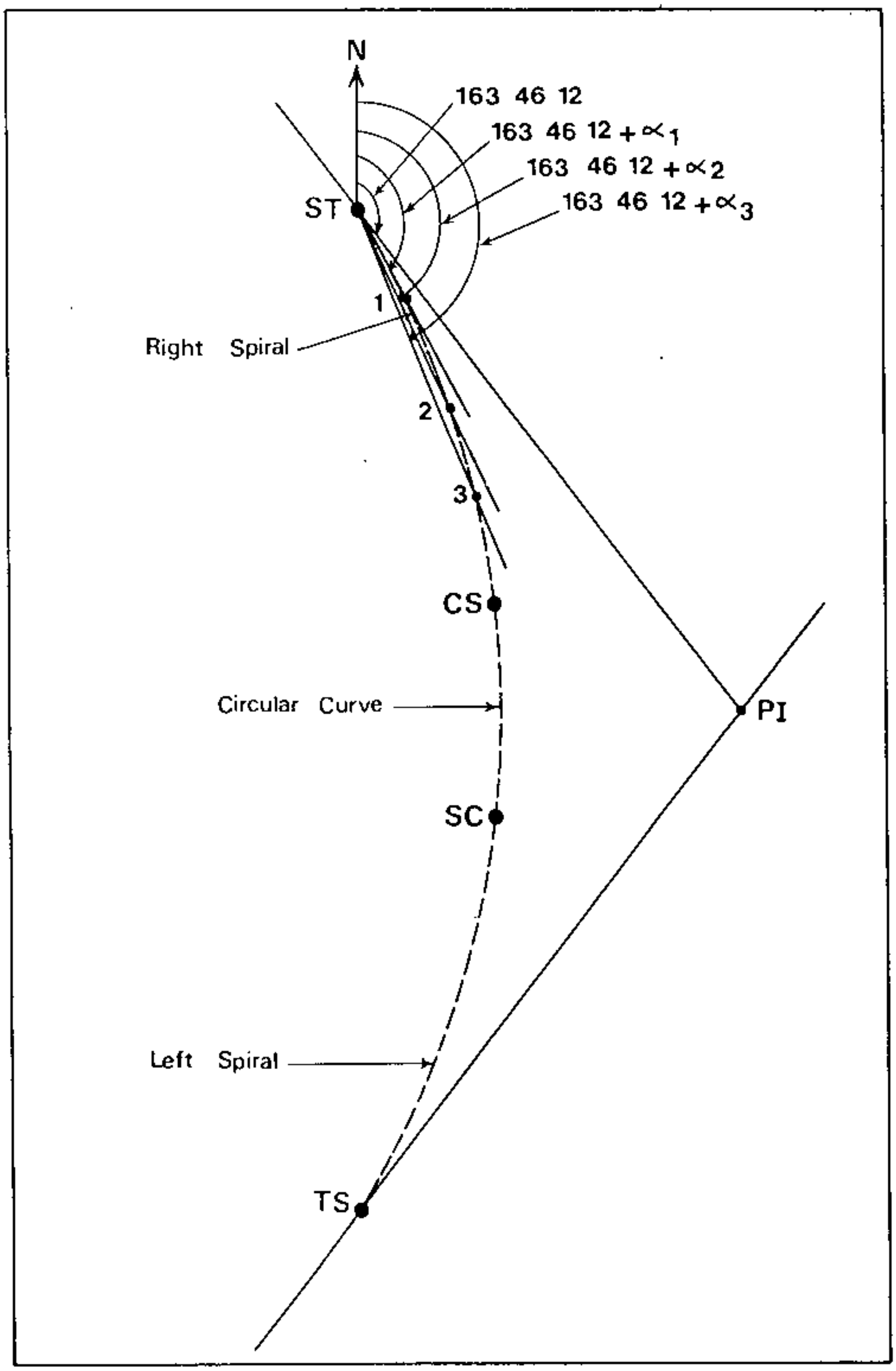


Figure (4-5): Right Spiral Of Example 1 (Part 2) d- Compute absolute coordinates for a group of points on the right spiral,

$X_1 = X_{ST} + s_1 * \sin (Az\ Of\ ST{-}{-}1)$

$= 87032.608 + 14.999 \sin (163\ 47\ 34.5) = 87036.794$

$Y_1 = Y_{ST} + s_1 * \cos (Az\ of\ ST{-}{-}1)$

$= 64972.48 + 14.999 \cos (163\ 47\ 34.5) = 64958.077$

$X_2 = 87032.608 + 29.999 \sin (163\ 51\ 56) = 87040.944$

$Y_2 = 64972.48 + 29.999 \cos (163\ 51\ 56) = 64943.663$

$X_3 = 87032.608 + 44.999 \sin (163\ 59\ 07) = 87045.023$

$Y_3 = 64972.48 + 44.999 \cos (163\ 59\ 07) = 64929.227$

$X_4 = 87032.608 + 59.999 \sin (164\ 09\ 04) = 87048.994$

$Y_4 = 64972.480 + 59.999 \cos (164\ 09\ 04) = 64914.760$

$X_5 = 87032.608 + 74.997 \sin (164\ 22\ 00) = 87052.818$

$Y_5 = 64972.480 + 74.997 \cos (164\ 22\ 00) = 64900.257$

$X_6 = 87032.608 + 89.992 \sin (164\ 37\ 46) = 87056.460$

$Y_6 = 64972.480 + 89.992 \cos (164\ 37\ 46) = 64885.707$

$X_7 = 87032.608 + 104.983 \sin (164\ 56\ 25) = 87059.886$

$Y_7 = 64972.480 + 104.983 \cos (164\ 56\ 25) = 64871.102$

$X_8 = 87032.608 + 119.966 \sin (165\ 17\ 51) = 87063.055$

$Y_8 = 64972.480 + 119.966 \cos (165\ 17\ 51) = 64856.442$

$X_9 = 87032.608 + 134.936 \sin (165\ 43\ 40) = 87065.916$

$Y_9 = 64972.480 + 134.936 \cos (165\ 43\ 40) = 64841.709$

$X_{10} = X_{CS} = X_{ST} + s_{10} \sin$ (Az of chord ST--CS)

$= 87032.608 + 149.896 \sin (166\ 09\ 25) = 87068.473$

$Y_{10} = Y_{CS} = Y_{ST} + s_{10} \cos$ (Az Of chord ST--CS)

$= 64972.608 + 149.896 \cos (166\ 09\ 25) = 64826.938$

As a check notice that the absolute coordinates of point CS are the same whether they have been computed from right spiral or from the circular curve.
The following table gives the final absolute coordinates for points on right spiral.

Table (4-7): Final Coordinates For Points On Right Spiral Of Example 1

Point No	X--Coordinate	Y--Coordinate
CS	87068.473	64826.938
9	87065.916	64841.709
8	87063.055	64856.442
7	87059.884	64871.102
6	87056.461	64885.707
5	87052.818	64900.527
4	87048.994	64914.760
3	87045.023	64929.227
2	87040.944	64943.663
1	87036.794	64958.077
(ST)	87032.688	64972.480

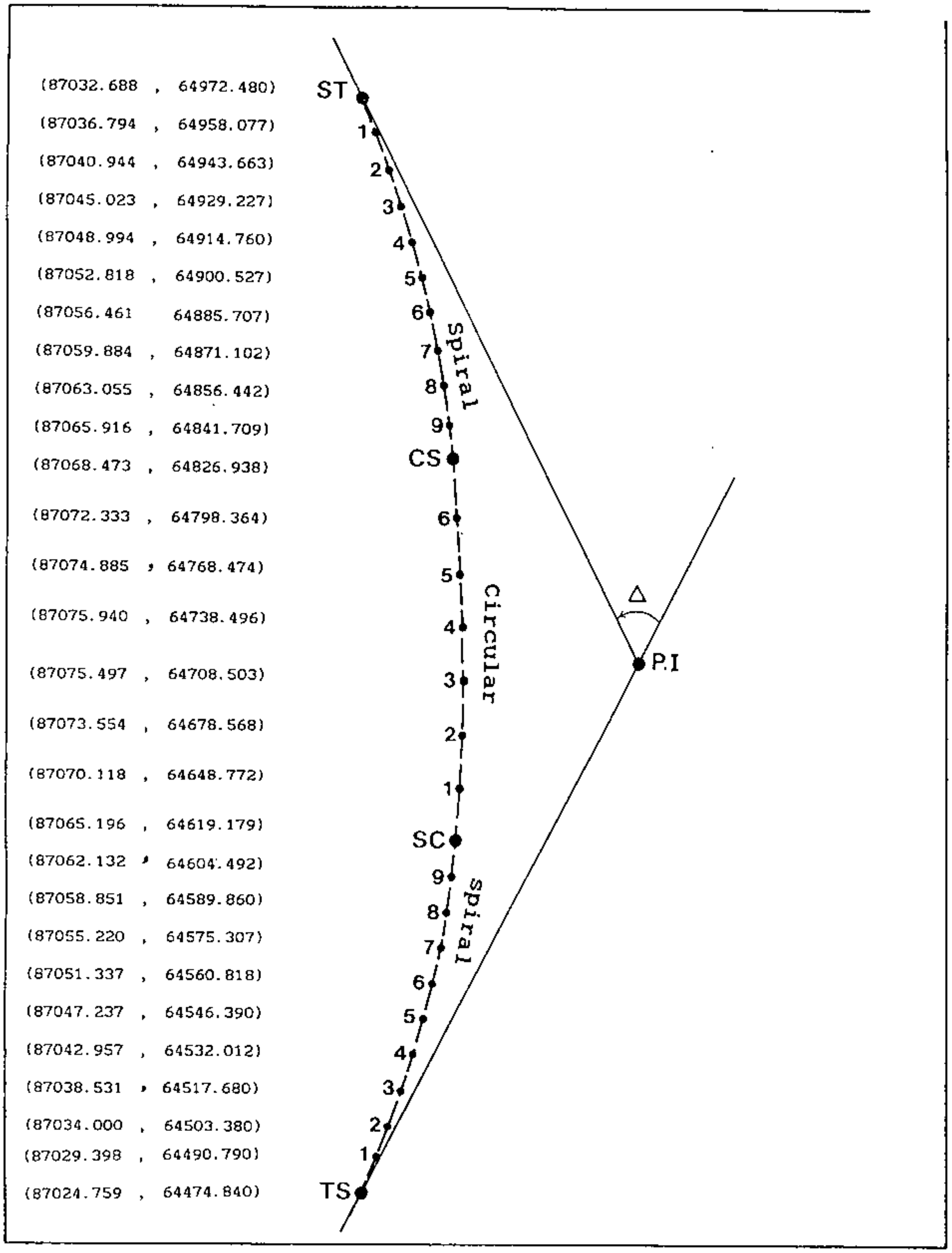


Figure (4-6): Final Absolute Coordinates For All Points Of The Equal-Tangent Spiraled Circular Curve Of Example 1 (Part 2)

Fig. 4-6 shows the location of all the points on the whole curve togather with their final absolute coordinates.

4-2 EXAMPLE 2

Design of 2- centered compound curve with one connecting spiral.

Given (Fig. 4-7):

a- X coordinate of PI = 1000 m.

b- Y coordinate of PI = 1000 m.

c- Azimuth of back tangent, A_{z1} = 45° 00´ 00´´

d- Azimuth of forward tangent, A_{z2} = 105° 00´ 00´´

e- Radius of left circular arc, R_1 = 200 m

f- Radius of right circular arc, R_2 = 400 m

g- Length of connecting spiral, L_s = 100 m

h- Angle between back tangent and the common tangent of the unshifted circular arcs at their point of tangency, Δ_1 = 30° 00´ 00´´

Required :

a- Compute elements of the curve.

b- Compute absolute coordinates of main points PC, C_1S, SC_2 & PT

c- Compute absolute coordinates for a group of points on the connecting spiral curve.

<u>Solution</u> :

1- Find central angles for each curve,

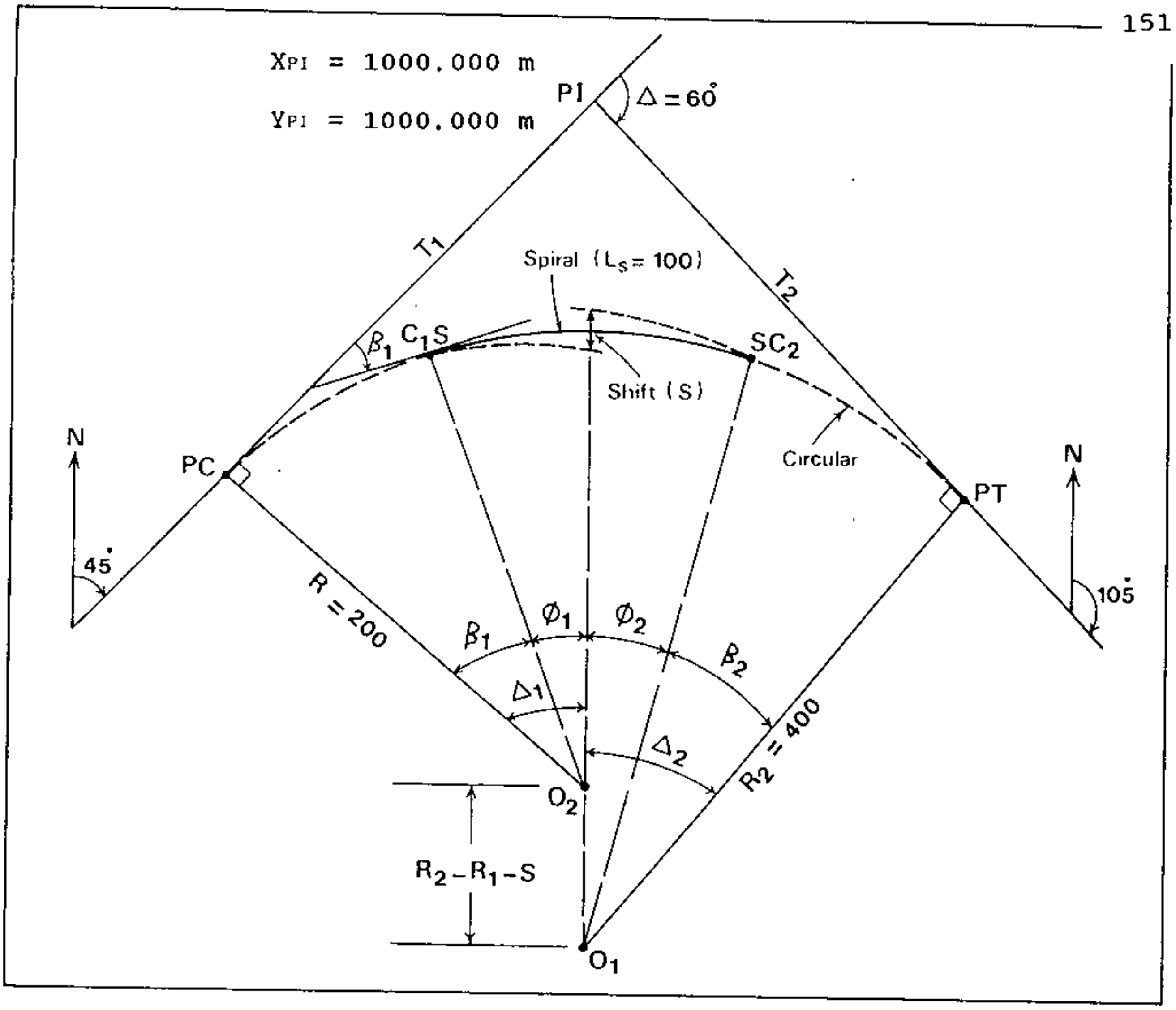


Figure (4-7): Two-Centered Compound Curve With 1 Spiral Of Example 2

$\Delta = Az_2 - Az_1 = 105\ \ 00\ \ 00 - 45\ \ 00\ \ 00$

$= 60^\circ\ 00'\ 00''$

$\Delta_2 = \Delta - \Delta_1 = 60\ 00\ 00 - 30\ 00\ 00 = 30^\circ\ 00'\ 00''$

$$\phi_1 = \frac{L_s}{2\ R_1} = \frac{100}{2*200} = 0.25 \text{ radian} = 14\ \ 19\ \ 26$$

$$\phi_2 = \frac{L_s}{2\ R_2} = \frac{100}{2*400} = 0.125 \text{ radian} = 07\ \ 09\ \ 43$$

Central angle of left circular arc= $\beta_1 = \Delta_1 - \phi_1$

$= 30\ 00\ 00 - 14\ 19\ 26$

$= 15\ \ 40\ \ 34$

Central angle of right circular arc=$\beta_2 = \Delta_2 - \phi_2$

$=30\ 00\ 00 - 07\ 09\ 43$

$= 22\ \ 50\ \ 17$

2- Compute the tangents lengths.

The shift S necessary to insert the spiral is computed. as follows

$$Y_1 = L_s\ (\frac{\phi_1}{3} - \frac{\phi_1^3}{42} + \frac{\phi_1^5}{1320})$$

$$Y_1 = 100*\ (\frac{0.25}{3} - \frac{0.25^3}{42} + \frac{0.25^5}{1320}) = 8.296 \text{ m}$$

similarly

$$Y_2 = 100*\ (\frac{0.125}{3} - \frac{0.125^3}{42} + \frac{0.125^5}{1320}) = 4.162 \text{ m}$$

$S_1 = Y_1 - R_1 \ (1 - \cos \phi_1)$

$= 8.296 - 200 \ (1 - \cos 14 \quad 19 \quad 26.2) = 2.078$ m

$S_2 = Y_2 - R_2 \ (1 - \cos \phi_2)$

$= 4.162 - 400 \ (1 - \cos 07 \quad 09 \quad 43 \ ) = 1.041$ m

The radial shift $S = S_1 - S_1$

$= 2.078 - 1.041 = 1.037$ m

The tangents lengths will be computed now by solving the traverse PI,PC,O_1,O_2,PT,PI (see Fig. 4-6),

$$T_2 = \frac{R_1 + (R_2 - R_1 - S) * \cos \Delta_1 - R_2 * \cos \Delta}{\sin \Delta}$$

$$= \frac{200+(400-200-1.037)*\cos 30 \ 0 \ 0 - 400*\cos 60 \ 0 \ 0}{\sin 60 \ 0 \ 0}$$

$= 198.96$ m

$T_1 = R_2 \sin \Delta - T_2 \cos \Delta - (R_2 - R_1 - S) \sin \Delta_1$

$= 400*\sin 60 - 198.96*\cos 60 - (400-200-1.037) \sin 60$

$= 147.450$ m

3- Compute the lengths of circular arcs,

$L_1 = \pi * R_1 * \beta_1/180 = \pi * 200 * 15 \quad 40 \quad 34/180$

$= 54.72$ m

$L_2 = \pi * R_2 * \beta_2/180 = \pi * 400 * 22 \quad 50 \quad 17/180$

$= 159.44$ m

4- Compute absolute coordinates of points PC & C_1S

$X_{PC} = X_{PI} + T_1 \sin (A_{z1} + 180)$

$Y_{PC} = Y_{PI} + T_1 \cos (A_{z1} + 180)$

$X_{PC} = 1000 + 147.45 * \sin (45\ 00\ 00 + 180) = 895.740$

$Y_{PC} = 1000 + 147.45 * \cos (45\ 00\ 00 + 180) = 895.740$

The length of chord PC--C_1S $= 2 * R_1 * \sin (\beta_1/2)$

$= 2*200 * \sin (15\ 40\ 34\ /2)$

$= 54.549$ m

The azimuth of chord PC--C_1S= $A_{Z1} + \beta_1/2$

$= 45\ 00\ 00 +(15\ 40\ 34\ /2)$

$= 52° \ 50' \ 17''$

Therefore

$X_{C1S} = 895.740 + 54.549 * \sin (52\ 50\ 17) = 939.210$

$Y_{C1S} = 895.740 + 54.549 * \cos (52\ 50\ 17) = 928.690$

5- Compute absolute coordinates of points on the connecting spiral.

Let us first compute local coordinates of the end point of the spiral SC_2, with respect to beginning point C_1S. The point C_1S will be the origin and the tangent at C_1S is the X--axis .

By solving the traverse C_1S,V,SC_2,O_2,O_1,C_1S shown in Fig. 4-7, we obtain

$x = R_2 \sin (\phi_1 + \phi_2) - (R_2 - R_1 - S) \sin \phi_1$

$=400*\sin(14\ 19\ 26+7\ 9\ 43)+(400-200-1.037)*\sin 14\ 19\ 26$

$= 97.28$ m

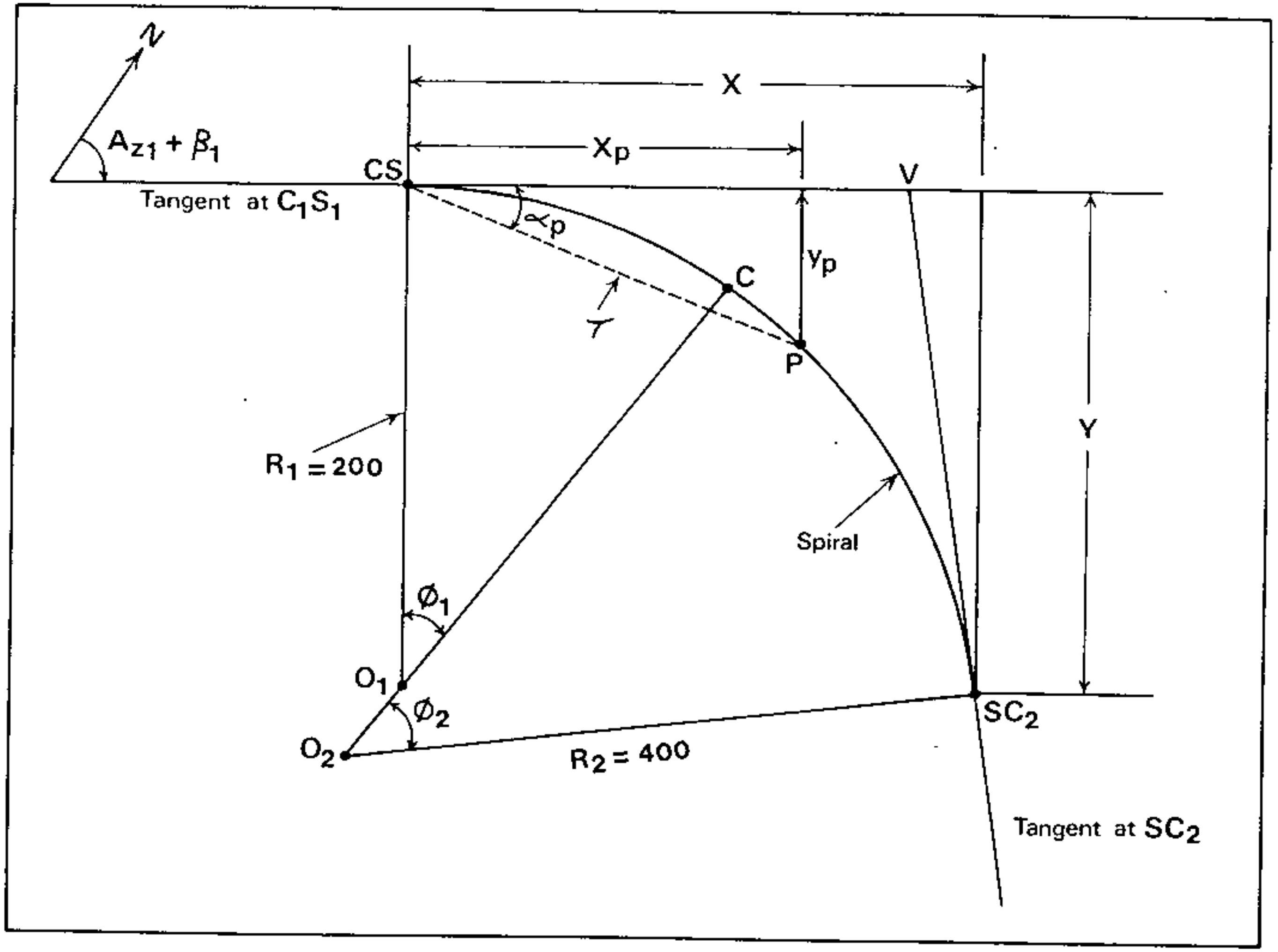


Table (4-8): Final Coordinates Of Points On Combining Spiral Of Example 2.

$y = R_1 - R_2 \cos(\phi_1 + \phi_2) + (R_2 - R_1 - S) \cos \phi_1$

=200–400*cos(14 19 26+07 09 43)

+(400–200–1.037)*cos 14 19 26

= 20.57 m

The length of chord C_1S-SC_2 = d =$(x^2 + y^2)^{0.5}$ = 99.43 m

$\lambda = \tan^{-1} y/x = \tan^{-1} 20.57/97.28 = 11° 56' 24''$

The Az.of chord C_1S--SC_2 = $Az_1 + \beta_1 + \lambda$

= 45 0 0 + 15 40 34 + 11 56 24

= 72° 36´ 58´´

Consequently, the absolute coordinates of point SC_2 are as follows,

$X_{SC2} = X_{C1S}$ + d sin (Az of chord C_1S--SC_2)

$Y_{SC2} = Y_{C1S}$ + d cos (Az of chord C_1S--SC_2)

X_{SC2} = 939.21 + 99.43*sin (72 36 58) = 1034.100

Y_{SC2} = 928.69 + 99.43*cos (72 36 58) = 958.400

As a check, let us compute the absolute coordinates of point SC_2 backward, starting from point PT.

$X_{PT} = X_{PI} + T_2 \sin(Az_2)$

= 1000 + 198.96*sin(105 00 00) = 1192.18

$Y_{PT} = Y_{PI} + T_2 \cos(Az_2)$

= 1000 + 198.96*cos(105 00 00) = 948.51

The chord PT--SC2 = d = 2 * R2 * sin $\beta_2/2$

= 2 * 400* sin(22 50 17/2)

=158.3859

The Az.of chord PT--SC2 = Az2 + 180 – $\beta_2/2$

= 105 0 0 + 180 – (22 50 17)/2

= 273° 34´ 52´´

Therefore the absolute coordinates of point SC2 are

$X_{SC2} = X_{PT} + d * \sin$ (Az of chord PT--SC2)

$Y_{SC2} = Y_{PT} + d * \cos$ (Az of chord PT--SC2)

X_{SC2} = 1192.18 + 158.3859* sin (273 34 52)= 1034.100

Y_{SC2} = 948.510 + 158.3859* cos (273 34 52)= 958.400

It can be seen that these two values are exactly the same as those obtained from left direction (from C1S)

In order to compute coordinates for a group of points on the combining spiral,use the equations (2.41) and (4.42) as shown below:

$$y = l^2*(\frac{D_1}{30})*(\frac{1}{2} + \frac{1}{3}k) - \frac{1}{6}*(\frac{D_1}{30})^3 * l^4 *(\frac{1}{4} + \frac{6}{10}k + \frac{1}{2}k^2 + \frac{1}{7}k^3)$$

$$x = l - \frac{1}{2}*(\frac{D_1}{30})^2 * l^3 * (\frac{1}{3} + \frac{1}{2}k + \frac{1}{5}k^2) + \frac{1}{24}*(\frac{D_1}{30})^4 * l^5 * (\frac{1}{5} + \frac{2}{3}k + \frac{6}{7}k^2 + \frac{1}{2}k^3 + \frac{1}{9}k^4)$$

where

x : The distance measured along the tangent at C_1S

y : The offset measured from tangent at C_1S

D_1 : Degree of curve for left circular arc = $\frac{30}{R_1}$

D_2 : Degree of curve of right circular arc = $\frac{30}{R_2}$

l : The distance along the spiral to any point P on the curve

$$k = \frac{1}{2} * \left(\frac{D_2-D_1}{D_1}\right) * \frac{1}{L_s}$$

If the spiral is divided into 10 equal parts then,

$l_1 = 10$, $l_2 = 20$, $l_3 = 30$,.........$l_{10} = 100$ m

$D_1 = \frac{30}{200} = 0.15$ rad , $D_2 = \frac{30}{400} = 0.075$ rad

$$K = \frac{1}{2} * \left(\frac{0.075-0.15}{0.15}\right) * \frac{1}{100}$$

K = − 0.0025 * l

Therefore

For $l_1 = 10$, K = − 0.025

For $l_2 = 20$, K = − 0.050

For $l_3 = 30$, K = − 0.075 ....etc

By substituting the above K values in the equations of x & y we get

For $l_1 = 10$,

$$x_1 = 10 - \frac{1}{2} * (\frac{0.15}{30})^2 * 10^3 * (\frac{1}{3} - \frac{1}{2} * 0.025 + \frac{1}{5} * 0.025^2)$$

$$+ \frac{1}{24} * (\frac{0.15}{30})^4 * 10^5 * (\frac{1}{5} - \frac{2}{3} * 0.025 + \frac{6}{7} * 0.025^2$$

$$- \frac{1}{2} * 0.025^3 + \frac{1}{9} * 0.025^4)$$

$$= 9.996 \text{ m}$$

$$y_1 = 10^2 * (\frac{0.15}{30}) * (\frac{1}{2} - \frac{1}{3} * 0.025) - \frac{1}{6} * (\frac{0.15}{30})^3 * 10^4 * (\frac{1}{4}$$

$$- \frac{6}{10} * 0.025 + \frac{1}{2} * 0.025^2 - \frac{1}{7} * 0.025^3)$$

$$= 0.245$$

The chord C_1S--1 = d = $(x_1^2 + y_1^2)^{0.5}$ = 9.999 m

The azimuth of chord C_1S--1 is found as follows

$$\alpha = \tan^{-1} \frac{y_1}{x_1} = \frac{0.245}{9.996} = 01° \ 24' \ 14''$$

The Az. of chord C_1S-1 = $Az_1 + \beta_1 + \alpha$

= 45 0 0 + 15 40 34 + 01 24 14

= 62 04 48

Therefore the absolute coordinates of point 1 on the combining spiral are finally computed as follows:

$X_1 = X_{C1S}$ + d * sin (Az of chord C_1S--1)

= 939.21 + 9.999 * sin (62 04 48)

= 948.045

$Y_1 = Y_{C1S} + d * \cos$ (Az of chord C_1S--1)

= 928.69 + 9.999 * cos (62 04 48)

= 933.370

The same procedure will be used to compute the absolute coordinates for points 2,3,4,5,.....10 on the combining spiral.

For this purpose the following table is prepared :

Table (4-8): Final Coordinates Of Points On Combining Spiral Of Example 2

pt	x	y	d	α	Az	X-coord	Y-coord
C_1S	0	0	0	0	60.676	939.210	928.690
1	9.999	0.245	9.999	1.404	62.080	948.045	933.370
2	19.97	0.97	19.99	2.781	63.547	957.090	937.620
3	29.9	2.13	29.98	4.075	64.751	966.330	941.480
4	39.77	3.72	39.94	5.344	66.020	975.700	944.920
5	49.57	5.71	49.89	6.571	67.247	985.220	947.990
6	59.29	8.05	59.83	7.732	68.408	994.84	950.710
7	68.92	10.74	69.75	8.857	69.533	1004.560	953.080
8	78.46	13.74	79.65	9.933	70.609	1014.340	955.140
9	87.90	17.02	89.53	10.96	71.635	1024.180	956.900
SC_2	97.26	20.56	99.40	11.94	72.612	1034.090	958.400

CHAPTER 5

THE COMPUTER PROGRAM

5-1 OBJECTIVES OF THE COMPUTER PROGRAM

The mathematical models presented in Chapter 3 will be converted in this chapter to a computer program that will carry out the following tasks :

1- Make a full traverse adjustment for the connecting traverse that links the points of intersections of any route. This will include :

a- Azimuth adjustment of all traverse sides.

b- Computation of preliminay coordinates of all PIs.

c- Computation of closure error for both azimuth and coordinates, and computation of relative and linear error of closure for coordinates.

d- Computation of final adjusted absolute coordinates of all points of intersections.

e- Computation of final adjusted length and azimuth of each course.

f- Computation of deflection angle at each PI.

Note that the type of ground control available for either the coordinates or azimuth, should be established in advance, in order to proceed with the adjustment process. Where the coordinates of a series of PI_s are already adjusted and given, then no adjustment will be needed for either the azimuth or the coordinates. For this case the program will compute directly the length of course , and the back and forward tangent at each PI.

2- Computation of the elements of all types of horizontal curves presented in Chapter 3. These elements include tangent length, shift, curve length........etc.

3- Computation of the absolute coordinates for points on any type of horizontal curve. The coordinates are first computed for the main points (e.g. TS, SC, CS..), then the coordinates for a group of points on each component of the whole curve are computed. For the circular curve the points are spaced at R/20 m, where for the spiral curves, they are spaced at R/40.

To design any type of horizontal curve, the following data should be given or computed beforehand:

a- Absolute coordinates of PI (X_{PI},Y_{PI}).

b- Azimuth of back tangent (A_{z1}).

c- Azimuth of forward tangent (A_{z2}).

d- Deflection angle (Δ).

Depending on the type of curve desired, other pertinent data should be input to the computer such as radii, the spiral length, ...etc.

4- Computation of the azimuth and length of the line that joins the beginning point on the curve (PC or TS) to any other point on the curve. This will serve as a supplementary aid to the surveyor.

5-2 HOW THE COMPUTER RROGRAM WORKS

The way in which the computer program works is illustrated diagrammatically by means of a flow chart shown in Fig. 5-1. The program is self-explanatory and easy to use. Furthermore, the program is flexible and expandable, so that it can be modified according to the user need.

5-3 APPLICATIONS ON THE COMPUTER PROGRAM

The output presented in Appendix B is obtained by running the program several times in order to get an example on each individual case presented in Chapter 3.

The following examples are given :

1- Traverse Adjustment: Example 1 (part one) that was done manualy in Chapter 4, is solved by the computer program in Appendix. It can be noted that the computer output gives identical results.
2- Equal-Tangent Spiraled Circular Curve: Example 1 (part two) in Chapter 4 is repeated by the computer program as

shown in Appendix. Identical results are also obtained.

3- Unequal-Tangent Spiraled Circular Curve.

4- Spiral applied to an Existing Circular Curve.

5- Equal-Tangent Double Spiral Curve.

6- Unequal-Tangent Double Spiral Curve.

7- Simple Circular Curve.

8- Two-Centered Compound Curve.

9- Three-Centered Compound Curve.

10- Two-Centered Compound Curve with One connecting Spiral: Example 2 that was solved manualy in Chapter 4 was carried out again by the computer program, as shown in Appendix . The same results are obtained.

11- Two-Centered Compound Curve with Three Spirals.

12- Simple Reverse Curve.

13- Spiraled Reverse Curve.

5-4 GENERAL REMARKS

1- For all types of curves, the absolute coordinates of the point of intersection PI, should be given. These coordinates can be either given by the program itself (traverse adjustment process), or they must be given externally when it is applied to an individual case of curves.

2- The deflection angle Δ, for each curve is computed with the knowledge of the azimuths of the back and forward tangents. Once again, these azimuths will be given either by the program itself (traverse adjustment process),or they must be given externally for individual case of curves.

3- The length of spiral curve may be given directly, or indirectly by inputting the design speed and the rate of change of radial acceleration.

4- All units are in meters.

5- If the azimuth of the back tangent is smaller than that of the forward tangent, then the azimuth of the line that joins the beginning point of the curve (PC or TS) to any other point on the curve, will increase gradually while proceeding along the curve and vice versa. This be defined by the computer program.

6- Absolute coordinates for the central points of circular arcs are computed by the program. These coordinates may be used when obstacles block the line of sight from the PC to the points on the curve.

7- For the simple reverse curves, two cases are dealt with :

a- The two opposing circular arcs have one common radius.

b- The two opposing circular arcs have different radii.

8- For the spiraled reverse curves, two cases are considered:

a- The 2 spirals of right part of the reverse curve are equal in length ($L_{s3} = L_{s4}$).

b- The 2 spirals of the right part of the reverse curve are not equal in length ($L_{s3} \neq L_{s4}$).

In both cases, the length of two spirals located on the left side of the the curve could be either equal or not.

9- The basis on which the values of the radius or degree of curve, and the spiral length is selected will depend on the geometric design criteria of design, of the highway standards.

CHAPTER 6

SUMMARY AND CONCLUSIONS

The following summary and general conclusions may be drawn from this study :

1- The computer program that has been developed in Chapter 5 serves as an aid to the design process of these curves by the coordinates method.

2- The design of horizontal curves by the coordinates method, gives more flexibility to the surveyors when they set out the route in the field. This means that the curve points can be set out either by the traditional method using the theodolite or by using the TSI. It means also that the curve can be setout starting from any point whose coordinates are already established. This point could be the PC (TS), PT (ST) , PI , the center point of circular arc, or any other point of known coordinates.

3- The design of horizontal curves by coordinates is an exact procedure that gives an accurate location of any point along the desired curve.

4- When the horizontal curve is designed by the coordinates method, then the use of the TSI will be the best method for setting out this curve, because it leads to more precision, speed and economy. This is in particular true in the case of spiraled reverse and spiraled compound curves.

5- In addition to computation of the coordinates of curve points, the computer program will give all the informat-ion that may be needed for setting out of curve. This includes all curve parameters such as curve length, tangent distance, shift, external distance,.......ets. It gives also the azimuth (WCB) of the line of sight that connects the first point on the curve (PC or TS) to any other points on the curve.

6- This study had covered many types of complicated spiral curves, such as the unequal-tangent spiraled circular curves, the unequal-tangent double spiral curves, the spiraled compound curves and the spiraled reverse curves. These types of curves are occasionally needed in the design of horizontal alinement of highways and railways.

7- The computer program that was developed in this thesis can serve as a substitute for some other programs that are available in the market. This program is designed in such a way that it suits the country needs and conforms to its specifications. Moreover, this computer program encompasses variety of cases and outputs, which are rarely included in other computer programs.

8- The authors hopes that this research will be helpful and useful to all those who are interested in this field, especially to Jordanian civil and surveying engineers, and surveyors.

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EXAMPLE 1

****
TRAVERSE ADJUSTMENT
****

INPUT:

Number of traverse points = 8

Two control points are given at start and end of traverse.

Enter coordinates of control points 0,1,N and N+1

Coord. of control point "0" = (86233.68 , 63961.22)

Coord. of control point "1" = (86005.65 , 63521.79)

Coord. of control point "N" = (88812.31 , 67718.06)

Coord. of control point "N+1" = (87766.35 , 67819.97)

Enter Length of Traverse Sides

Length of Side 1 = 708.07

Length of Side 2 = 696.21

Length of Side 3 = 556.81

Length of Side 4 = 771.76

Length of Side 5 = 1028.39

Length of Side 6 = 1076.82

Length of Side 7 = 866.77

Enter Clockwise Angles at Points of Traverse

Angle at Point 1 = 61 44 00

Angle at Point 2 = 109 13 00

Angle at Point 3 = 179 40 20

Angle at Point 4 = 145 44 10

Angle at Point 5 = 237 52 30

Angle at Point 6 = 183 00 49

Angle at Point 7 = 169 10 30

Angle at Point 8 = 61 44 19

OUTPUT:

Azimuth of first control side "0--1" = 207 25 30

Azimuth of last control side "N--N+1" = 275 33 52

Closure error in Azimuth = 00 01 17

Table (1-a): Computer run showing adjusted azimuth for an example on connecting traverse

ADJUSTED AZIMUTHS			
Azimuth of Side 1 =	89	09	20
Azimuth of Side 2 =	18	22	11
Azimuth of Side 3 =	18	02	21
Azimuth of Side 4 =	343	46	21
Azimuth of Side 5 =	41	38	41
Azimuth of Side 6 =	44	39	22
Azimuth of Side 7 =	33	49	42
Azimuth of Side 8 =	275	33	52

Table (1-b): Computer run showing preliminary coordinates

PRELIMINARY COORDINATES

X1 = 86005.65
Y1 = 63521.79

X2 = 86713.46
Y2 = 63532.23

X3 = 86933.05
Y3 = 64192.96

X4 = 87105.48
Y4 = 64722.40

X5 = 86889.81
Y5 = 65463.41

X6 = 87573.19
Y6 = 66231.90

X7 = 88330.03
Y7 = 66997.88

X8 = 88812.57
Y8 = 67717.91

Closure error in X--Coordinate = 0.258

Closure error in Y--Coordinate = -0.148

Linear error = 0.297

Relative error = 0.00005215

Table (1-c): Computer run showing adjusted coordinates.

FINAL ADJUSTED COORDINATES

X1 = 86005.56
Y1 = 63521.79

X2 = 86713.61
Y2 = 63532.24

X3 = 86932.99
Y3 = 63192.99

X4 = 87105.39
Y4 = 64722.45

X5 = 86889.69
Y5 = 65463.48

X6 = 87573.02
Y6 = 66232.00

X7 = 88329.81
Y7 = 66998.01

X8 = 88812.31
Y8 = 67718.06

Table (1-d): Computer run showing final length and azimuths of traverse sides.

FINAL LENGTH AND AZIMUTH OF SIDES
Length of side 1 = 708.038
Azimuth of side 1 = 89 09 14
Length of side 2 = 696.215
Azimuth of side 2 = 18 21 59
Length of side 3 = 556.82
Azimuth of side 3 = 18 02 12
Length of side 4 = 771.787
Azimuth of side 4 = 343 46 13
Length of side 5 = 1028.377
Azimuth of side 5 = 41 38 30
Length of side 6 = 1076.805
Azimuth of side 6 = 44 39 12
Length of side 7 = 866.767
Azimuth of side 7 = 33 49 32

Table (1-e): Computer run showing deflection angles at PIs.

DEFLECTION ANGLES AT POINTS OF INTERSECTIONS
Deflection Angle at PI 2 = 70 47 14
Deflection Angle at PI 3 = 00 19 47
Deflection Angle at PI 4 = 325 44 01
Deflection Angle at PI 5 = 302 07 43
Deflection Angle at PI 6 = 03 00 41
Deflection Angle at PI 7 = 10 49 40

EXAMPLE 2

****
DESIGN OF EQUAL-TANGENT SPIRALED CIRCULAR CURVE
****

INPUT:

X--Coordinate of PI = 87105.39 m

Y--Coordinate of PI = 64722.35 m

Deflection angle at PI = 43 16 02

Azimuth of back tangent = 18 02 12

Azimuth of forward tangent = 343 46 10

Length of spiral = 150 m

Radius of circular arc = 600 m

OUTPUT:

Degree of curve = 02 51 53

Tangent length = 260.4119 m

Spiral central angle = 07 09 43

Shift of the circular arc = 1.5616 m

The distance X along tangent to point SC = 149.7658 m

The offset Y from tangent to point SC = 6.243 m

The distance X_0 from shifted PC to TS = 74.961 m

Central angle of circular arc = 19 56 35

Length of circular arc = 208.846 m

External distance = 29.489 m

Table (2-a): Computer run showing coordinates for group of points on Equal-Tangent Spiraled Circular curve.

POINT NO	X--COORDINATE	Y--COORDINATE
TS	**87024.76**	**64474.84**
1	87029.40	64489.10
2	87034.00	64503.38
3	87038.53	64517.68
4	87042.96	64532.01
5	87047.24	64546.39
6	87051.34	64560.82
7	87055.22	64575.30
8	87058.85	64589.86
9	87062.19	64604.48
10	87065.19	64619.18
SC	**87065.19**	**64619.18**
12	87070.12	64648.77
13	87073.56	64678.57
14	87075.49	64708.50
15	87075.94	64738.49
16	87074.88	64768.48
17	87072.33	64798.36
CS	**87068.48**	**64826.94**
19	87065.93	64841.73
20	87063.06	64856.45
21	87059.89	64871.11
22	87056.46	64885.71

23	87052.82	64900.26
24	87048.99	64914.77
25	87045.02	64929.23
26	87040.94	64943.67
27	87036.80	64958.08
ST	**87032.61**	**64972.49**

Table (2-b): Computer run showing the length and azimuth of the line joining TS to any point on curve.

POINT NO	AZIMUTH	DISTANCE
1	18 01 11	15.001
2	17 56 30	30.002
3	17 49 23	45.000
4	17 39 14	59.997
5	17 26 21	74.998
6	17 10 41	89.991
7	16 52 02	104.981
8	16 30 38	119.966
9	16 06 14	134.940
10	15 39 01	149.897
11	15 39 01	149.897
12	14 37 00	179.747
13	13 28 10	209.493
14	12 05 00	239.109
15	10 59 07	268.578

16	09	41	14	297.884
17	08	21	53	327.002
18	07	04	40	354.809
19	06	24	10	369.194
20	05	43	50	383.526
21	05	03	59	397.828
22	04	24	43	412.096
23	03	46	26	426.350
24	03	09	11	440.597
25	02	33	13	454.846
26	01	58	38	469.108
27	01	25	37	483.392
28	00	54	14	497.710

EXAMPLE 3

```
*************************************************
DESIGN OF UNEQUAL-TANGENT SPIRALED CIRCULAR CURVE
*************************************************

INPUT:

X--Coordinate of PI = 1000 m

Y--Coordinate of PI = 1000 m

Deflection angle at PI =  50  00  00

Azimuth of back tangent = 45  00  00

Azimuth of forward tangent = 95  00  00

Radius of circular curve = 400 m

Length of left spiral = 100 m

Length of right spiral = 150 m
---------------------------------------------


OUTPUT:

Left spiral central angle = 07  09  43

Right spiral central angle = 10  44  34

Length of circular arc = 224.0659 m

Back tangent length = 238.679 m

Forward tangent length = 260.830 m

Distance X along back tangent to point SC = 99.844 m

Distance X along forward tangent to point CS = 149.474 m

Offset from back tangent to point SC = 4.162 m

Offset from forward tangent to point CS = 9.351 m
----------------------------------------------------------
```

Table (3-a): Computer run showing coordinates for group of points on Unequal-Tangent Spiraled Circular curve.

POINT NO	X--COORDINATE	Y--COORDINATE
TS	**831.228**	**831.228**
1	838.302	838.296
2	845.394	845.347
3	852.521	852.362
4	859.700	859.323
5	866.948	866.212
6	874.282	873.010
7	881.717	879.697
8	889.268	886.253
9	896.949	892.656
SC	**904.772**	**898.886**
11	920.867	910.754
12	937.535	921.804
13	954.734	932.006
14	972.422	941.337
15	990.554	949.771
16	1009.058	957.289
17	1027.969	963.871
18	1047.158	969.501
19	1066.260	974.165
20	1086.260	977.852
21	1106.075	980.551
CS	**1409.557**	**973.556**
23	1120.079	981.858

24	1130.058	982.504
25	1140.048	982.934
26	1150.046	983.164
27	1160.045	983.211
28	1170.045	983.091
29	1180.041	982.821
30	1190.033	982.418
31	1200.019	981.898
32	1210.000	981.279
33	1219.975	980.576
34	1229.945	979.807
35	1239.912	978.988
36	1249.876	978.136
ST	**1259.838**	**977.267**

Table (3-b): Computer run showing the length and azimuth of the line joining TS to any point on curve.

POINT NO	AZIMUTH	DISTANCE
1	45 01 26	10.000
2	45 05 43	20.000
3	45 12 53	30.000
4	45 22 54	39.999
5	45 35 48	49.998
6	45 51 33	59.995
7	46 10 11	69.988
8	46 31 40	79.977
9	46 56 00	89.959

10	47	23	13	99.931
11	48	25	15	119.831
12	49	34	05	139.660
13	50	47	10	159.405
14	52	03	05	179.052
15	53	20	59	198.588
16	54	40	18	218.001
17	56	00	43	237.278
18	57	21	57	256.408
19	58	43	51	275.378
20	60	06	15	294.176
21	61	29	05	312.790
22	62	10	27	595.585
23	62	27	32	325.767
24	63	09	00	334.938
25	63	50	15	344.070
26	64	31	09	353.170
27	65	11	35	362.242
28	65	51	26	371.293
29	66	30	37	380.330
30	67	09	03	389.357
31	67	46	38	398.382
32	68	23	20	407.410
33	68	59	03	416.448
34	69	33	44	425.501
35	70	07	20	434.575
36	70	39	48	443.675
37	71	11	04	452.806

EXAMPLE 4

```
******************************************
DESIGN OF EQUAL-TANGENT DOUBLE SPIRAL CURVE
******************************************
```

INPUT:

X-Coordinate of P1 = 1000 m

Y-Coordinate of P1 = 1000 m

Deflection Angle = 30 0 0

Azimuth of Back Tangent = 30 0 0

Azimuth of Forward Tangent = 60 0 0

Length of spiral curve = 170 m

OUTPUT:

The distance X along the tangent to point SS =168.836 m

The offset Y to point SS = 14.763

The spiral central angle = 15 0 0

The tangent length = 172.7942 m

Radius at common point SS = 324.676 m

External distance = 15.283 m

Table (4-a): Computer run showing coordinates for group of points on Equal-Tangent Double Spiral curve.

Point No	X--Coordinate	Y--Coordinate
TS	**913.603**	**850.356**
1	917.604	857.283
2	921.614	864.206
3	925.639	871.120
4	929.689	878.019
5	933.770	884.900
6	937.891	891.756
7	942.060	898.584
8	946.284	905.378
9	950.571	912.132
10	954.928	918.842
11	959.363	925.500
12	963.882	932.101
13	968.493	938.639
14	973.201	945.106
15	978.014	951.496
16	982.938	957.802
17	987.977	964.015
18	993.138	970.127
19	998.426	976.131
20	1003.844	982.016
21	1009.397	987.774
SS	**1010.807**	**989.193**
23	1016.532	994.781

24	1022.387	1000.232
25	1028.361	1005.552
26	1034.447	1010.744
27	1040.636	1015.814
28	1046.919	1020.766
29	1053.289	1025.606
30	1059.737	1030.340
31	1066.258	1034.974
32	1072.844	1039.516
33	1079.489	1043.971
34	1086.186	1048.347
35	1092.930	1052.651
36	1099.714	1056.890
37	1106.534	1061.072
38	1113.384	1065.204
39	1120.259	1069.295
40	1127.154	1073.351
41	1134.065	1077.382
42	1140.985	1081.395
ST	**1149.644**	**1086.397**

Table (4-b): Computer run showing the length and azimuth of the line joining TS to any point on curve.

Point No	Azimuth	Distance
1	30 0 39	8.000
2	30 2 39	16.000
3	30 5 58	24.000
4	30 10 37	32.000
5	30 16 36	40.000
6	30 23 54	47.999
7	30 32 33	55.998
8	30 42 31	63.996
9	30 53 48	71.993
10	31 6 26	79.988
11	31 20 23	87.981
12	31 35 39	95.970
13	31 52 16	103.956
14	32 10 12	111.936
15	32 29 27	119.909
16	32 50 2	127.875
17	33 11 57	135.830
18	33 35 11	143.774
19	33 59 44	151.704
20	35 54 45	134.455
21	36 27 22	141.332
22	37 0 2	148.195
23	37 32 37	155.046
24	38 5 0	161.885

25	38	37	6	168.713
26	39	8	49	175.530
27	39	40	6	182.338
28	40	10	53	189.138
29	40	41	7	195.930
30	41	10	46	202.716
31	41	39	47	209.496
32	42	8	8	216.272
33	42	35	48	223.045
34	43	2	46	229.817
35	43	28	58	236.587
36	43	54	26	243.358
37	44	19	7	250.130
38	44	43	0	256.905
39	45	6	5	263.683
40	45	28	21	270.466
41	45	49	47	277.255
42	46	10	22	284.050
43	46	40	59	294.744

EXAMPLE 5

****
DESIGN OF UNEQUAL-TANGENT DOUBLE SPIRAL CURVE
****

INPUT:

X- Coordinate of P1 = 1000 m

Y-Coordinate of P1 = 1000 m

Deflection Angle = 30 0 0

Azimuth of Back Tangent = 30 0 0

Azimuth of Forward Tangent = 60 0 0

Length of left spiral = 100 m

Left spiral central angle = 10 0 0

OUTPUT:

Length of right spiral = 200 m

right spiral central angle = 20 0 0

The left tangent length = 135.779 m

The right tangent length = 169.228 m

The distance X along tangent to point SS = 99.695 m

The offset Y from tangent to point SS = 5.805 m

Table (5-a): Computer run showing coordinates for group of points on Unequal-Tangent Double Spiral curve.

Point No	X--Coordinate	Y--Coordinate
TS	**932.110**	**882.411**
1	935.612	888.473
2	939.124	894.528
3	942.657	900.571
4	946.221	906.596
5	949.825	912.596
6	953.481	918.566
7	957.199	924.497
8	960.986	930.384
9	964.854	936.218
10	968.812	941.992
11	972.867	947.697
12	977.030	953.325
13	981.306	958.866
14	985.705	964.311
SS	**986.986**	**965.848**
16	991.549	971.155
17	996.237	976.354
18	1001.042	981.444
19	1005.956	986.429
20	1010.975	991.309
21	1016.091	996.086
22	1021.298	1000.764
23	1026.590	1005.346
24	1031.963	1009.833

25	1037.709	1014.230
26	1042.925	1018.540
27	1048.505	1022.767
28	1054.144	1026.914
29	1059.838	1030.985
30	1065.582	1034.985
31	1071.373	1038.919
32	1077.205	1042.790
33	1083.076	1046.603
34	1088.980	1050.362
35	1094.915	1054.074
36	1100.877	1057.742
37	1106.863	1061.371
38	1112.869	1064.966
39	1118.893	1068.533
40	1124.930	1072.076
41	1130.977	1075.600
42	1137.033	1079.112
ST	1146.557	1084.615

Table (5-b): Computer run showing the length and azimuth of the line joining TS to any point on curve

Point No	Azimuth			Distane
1	30	00	58	7.000
2	30	03	55	14.000
3	30	08	48	21.000
4	30	15	40	28.000
5	30	24	29	34.999
6	30	35	16	41.998
7	30	48	01	48.996
8	31	02	43	55.993
9	31	19	22	62.987
10	31	37	59	69.977
11	31	58	34	76.963
12	32	21	06	83.943
13	32	45	35	90.916
14	33	12	02	97.878
15	33	19	56	99.865
16	33	48	48	106.811
17	34	19	05	113.743
18	34	50	22	120.661
19	35	22	21	127.565
20	34	25	37	159.618
21	34	52	49	167.512
22	34	59	49	169.483
23	35	28	36	177.350
24	35	58	22	185.194

25	36	28	50	193.017
26	36	59	46	200.818
27	37	30	57	208.599
28	38	02	13	216.362
29	38	33	25	224.108
30	39	04	27	231.839
31	39	35	10	239.557
32	40	05	30	247.263
33	40	35	21	254.960
34	41	04	39	262.650
35	41	33	20	270.335
36	42	01	21	278.017
37	42	28	37	285.698
38	42	55	07	293.381
39	43	20	48	301.066
40	43	45	38	308.757
41	44	09	34	316.456
42	44	32	34	324.163
43	45	00	00	333.813

EXAMPLE 6

****
DESIGN OF SPIRAL APPLIED TO EXISTING CIRCULAR CURVE
****

INPUT:

X-Coordinate of P1 = 1000 m

Y-Coordinate of P1 = 1000 m

Deflection Angle Δ = 25 0 0

Azimuth of Back Tangent = 40 0 0

Azimuth of Forward Tangent = 65 0 0

Radius of existing circular curve = 400 m

OUTPUT:

Spiral central angle = 7 30 0

Length of circular curve = 69.813 m

Radius of sharpened circular curve = 299.937 m

Length of spiral = 78.52356 m

The distance from the original PC to TS = 26.178 m

The distance along the tangent to point SC = 78.389 m

The offset distance from tangent to point SC = 3.420 m

The tangent length = 114.856 m

External distance = 9.711 m

Table (6-a): Computer run showing coordinates for group of points on the curve.

Point No	X--Coordinate	Y--Coordinate
TS	**926.172**	**912.015**
1	932.605	919.671
2	939.071	927.299
3	945.601	934.873
4	952.227	942.362
5	958.980	949.738
6	965.887	956.969
7	972.976	964.021
SC	**976.608**	**972.121**
9	991.685	985.259
10	1007.400	997.626
11	1023.713	1009.193
CS	**1032.983**	**1015.379**
13	1043.015	1017.548
14	1051.682	1022.535
15	1060.471	1027.306
16	1069.356	1031.895
17	1078.314	1036.337
18	1087.326	1040.671
ST	**1104.095**	**1048.540**

Table (6-b): Computer run showing the length and azimuth of the line joining TS to any point on curve.

Point No	Azimuth	Distance
1	40 2 25	10.000
2	40 9 44	20.000
3	40 21 53	29.999
4	40 38 55	39.998
5	41 0 48	49.994
6	41 27 34	59.984
7	41 59 11	69.966
8	40 0 2	78.464
9	41 48 40	98.268
10	43 29 42	118.014
11	45 6 25	137.688
12	45 56 22	148.637
13	47 54 41	157.447
14	48 38 2	167.235
15	49 21 18	176.998
16	50 3 44	186.743
17	50 44 46	196.478
18	51 23 53	206.212
19	52 30 0	224.268

EXAMPLE 7

****
DESIGN OF SIMPLE CIRCULAR CURVE
****

INPUT:

X-Coordinate of P1 = 1000 m

Y-Coordinate of P1 = 1000 m

Deflection Angle = 50 0 0

Azimuth of Back Tangent = 20 0 0

Azimuth of Forward Tangent = 70 0 0

Radius of curve = 400 m

OUTPUT:

Degree of curve = 4 17 49

Tangent length = 186.5231 m

Length of circular curve = 349.065 m

External distance = 41.351 m

Long Chord = 338.094 m

Mid-Ordinate distance = 37.476 m

The X--Coordinate of center Point 0 = 1073.013

The Y--Coordinate of center Point 0 = 1200.603

Table (7,a): Computer run showing coordinates for group of points on the Simple Circular Curve.

POINT NO	X--COORDINATE	Y--COORDINATE
PC	**936.205**	**824.726**
1	943.513	843.341
2	951.741	861.567
3	960.870	879.360
4	970.877	896.674
5	981.737	913.466
6	993.423	929.695
7	1005.905	945.319
8	1019.152	960.300
9	1033.132	974.600
10	1047.809	988.183
11	1063.146	1001.016
12	1079.105	1013.066
13	1095.647	1024.304
14	1112.730	1034.701
15	1130.311	1044.231
16	1148.346	1052.870
17	1166.791	1060.598
PT	**1175.274**	**1063.795**

Table (7-b): Computer run showing the length and azimuth of the line joining PC to any point on curve.

POINT NO	AZIMUTH	DISTANCE
1	21 25 56	19.998
2	22 51 53	39.983
3	24 17 49	59.944
4	25 43 46	79.867
5	27 9 43	99.740
6	28 35 39	119.551
7	30 01 36	139.286
8	31 27 32	158.935
9	32 53 29	178.485
10	34 19 26	197.923
11	35 45 22	217.238
12	37 11 19	236.416
13	38 37 16	255.447
14	40 03 12	274.318
15	41 29 09	293.018
16	42 55 05	311.535
17	44 21 02	329.857
18	45 00 00	338.095

EXAMPLE 8

****
DESIGN OF 2-CENTRED COMPOUND CURVE
****

INPUT:

Y-Coordinate of P1 = 1000 m

Y-Coordinate of P1 = 1000 m

Deflection Angle = 60 00 00

Azimuth of Back Tangent = 45 0 0

Azimuth of Forward Tangent = 105 0 0

R1 = 400 m

R2 = 600 m

Deflection angle of left arc Δ_1 = 30 00 00

OUTPUT:

Length of back tangent P1--T1 = 261.880 m

Length of common tangent Nj = 267.949 m

Length of Forward tangent P1--T2 = 315.470 m

Length of left arc = 209.439 m

Length of right arc = 314.159 m

Length of long Chord of Left Arc = 207.055 m

Length of Long Chord of Right Arc = 310.583 m

The X--Coordinates of Center Point 01 = 1097.665

The Y--Coordinated of Centre Point 01 = 1097.665

The X--Coordinates of Centre Point 02 = 579.556

The Y--Coordinates of Center Point 02 = -155.291

Table (8,a): Computer run showing coordinates for group of points on Two-Centered Compound curve.

POINT NO	X--COORDINATE	Y--COORDINATE
T1	**814.823**	**814.823**
1	829.312	828.606
2	844.473	842.647
3	860.266	853.914
4	876.653	865.377
5	893.592	876.006
6	911.041	885.776
7	928.957	894.661
8	947.294	902.640
9	966.007	909.692
10	985.050	915.800
K	**994.138**	**918.350**
12	1023.298	925.387
13	1052.773	930.958
14	1082.489	935.049
15	1112.373	937.650
16	1142.350	938.753
17	1172.344	938.357
18	1202.281	936.463
19	1232.086	933.074
20	1261.684	928.000
21	1291.002	921.853
T2	**1304.721**	**918.350**

Table (8-b): Computer run showing the length and azimuth of the line joining T_1 to any point on curve.

POINT NO	AZIMUTH	DISTANCE
1	46 25 56	19.998
2	47 51 53	39.983
3	49 17 49	59.944
4	50 43 46	79.867
5	52 09 43	99.740
6	53 35 39	119.551
7	55 01 36	139.286
8	56 27 32	158.936
9	57 53 29	178.485
10	59 19 26	197.923
11	60 00 00	207.055
12	62 03 38	235.980
13	63 59 04	264.778
14	65 48 43	293.428
15	67 34 10	321.905
16	69 16 27	350.190
17	70 56 18	378.262
18	72 34 04	406.104
19	74 10 38	433.696
20	75 45 48	461.020
21	77 19 55	488.06
22	78 04 03	500.718

EXAMPLE 9

```
*************************************
DESIGN OF 3-CENTERED COMPOUND CURVE
*************************************
```

INPUT:

Y-Coordinate of P1 = 1000 m

Y-Coordinate of P1 = 1000 m

Deflection Angle = 60 00 00

Azimuth of Back Tangent = 45 0 0

Azimuth of Forward Tangent = 105 0 0

R1 = 600 m

R2 = 400 m

R3 = 200 m

Δ_1 = 25 00 00

Δ_3 = 20 00 15

OUTPUT:

Δ_2 = 35 00 00

Δ_4 = 15 00 00

Length of back tangent PI-PC = 296.776 m

Length of first common tangent NJ = 247.255 m

Length of second common tangent KL = 96.861 m

Length of forward tangent PI-PT = 204.748 m

Length of first arc = 261.799 m

Length of second arc = 139.626 m

Length of third arc = 52.359 m

Coordinates of center point O_1 = (1214.412,365.884)

Coordinates of center point O_2 = (1036.808,575.877)

Coordinates of center point O_3 = (1146.008,753.822)

Table (9-a): Computer run showing coordinates for group of points on the Three-Centered Compound curve.

POINT NO	X--COORDINATE	Y--COORDINATE
PC	**790.148**	**790.148**
1	811.882	810.822
2	834.623	830.384
3	858.313	848.785
4	882.893	865.979
5	908.302	881.923
6	934.475	896.577
7	961.349	909.905
8	988.855	921.873
T1	**1009.200**	**929.873**
10	1028.157	936.067
11	1047.408	941.479
12	1066.906	945.923
13	1086.602	949.386
14	1106.446	951.861
15	1126.389	953.341
T2	**1146.008**	**953.822**
17	1156.004	953.572
18	1165.974	952.823
19	1175.895	951.576
20	1185.742	949.835
21	1195.489	947.605
PT	**1197.772**	**947.007**

Table (9-b): Computer run showing the length and azimuth of the line joining PC to any point on curve.

POINT NO	AZIMUTH	DISTANCE
1	46 25 56	29.997
2	47 51 53	59.976
3	49 17 49	89.916
4	50 43 46	119.800
5	52 09 43	149.610
6	53 35 39	179.326
7	55 01 36	208.930
8	56 27 32	238.403
9	57 30 00	259.728
10	58 29 17	279.179
11	59 32 02	298.470
12	60 37 36	317.587
13	61 45 28	336.515
14	62 55 14	355.241
15	64 06 38	373.752
16	65 18 01	391.696
17	65 55 48	400.697
18	66 35 41	409.523
19	67 17 29	418.163
20	68 01 03	426.608
21	68 46 15	434.849
22	68 57 09	436.763

EXAMPLE 10

****

DESIGN OF 2-CENTRED SPIRALED COMPOUND CURVE WITH 1 SPIRAL

****

INPUT:

X--Coordinate of PI = 1000 m

Y--Coordinate of PI = 1000 m

Deflection angle at PI = 60 00 00

Azimuth of back tangent = 45 00 00

Azimuth of forward tangent = 105 00 00

R_1 = 200 m

R_2 = 400 m

Length of connecting spiral = 100 m

Δ_1 = 30 00 00

OUTPUT:

Δ_2 = 30 00 00

Length of back tangent = 147.448 m

Length of forward tangent = 198.962 m

Length of left circular arc = 54.719 m

Length of right circular arc = 159.439 m

Table (10-a): Computer run showing coordinates for group of points on 2-Centered Compound with 1 spiral.

POINT NO	X--COORDINATE	Y-COORDINATE
PC	**895.739**	**895.739**
1	902.984	902.630
2	910.564	909.151
3	918.460	915.284
4	926.654	921.016
5	935.123	926.330
C1S	**939.211**	**928.690**
7	948.046	933.371
8	957.094	937.628
9	966.324	941.473
10	975.710	944.923
11	985.226	947.994
12	994.850	950.707
13	1004.564	953.081
14	1014.349	955.139
15	1024.192	956.902
SC2	**1034.106**	**958.397**
17	1053.979	960.629
18	1073.939	961.864
19	1093.935	962.100
20	1113.919	961.336
21	1133.839	959.574
22	1153.646	956.819
23	1173.291	953.078
PT	**1192.183**	**948.505**

Table (10-b): =Computer run showing the length and azimuth of the line joining PC to any point on curve.

POINT NO	AZIMUTH			DISTANCE
1	46	25	57	9.999
2	47	51	53	19.992
3	49	17	49	29.972
4	50	43	46	39.933
5	52	09	43	49.870
6	52	50	16	54.549
7	54	16	00	64.438
8	55	40	38	74.291
9	57	03	34	84.107
10	58	24	27	93.885
11	59	43	02	103.627
12	60	59	12	113.334
13	62	12	51	123.008
14	63	23	54	132.653
15	64	32	18	142.272
16	65	38	12	151.894
17	67	42	10	171.029
18	69	38	29	190.073
19	71	29	17	209.011
20	73	15	58	227.828
21	74	59	30	246.509
22	76	40	33	265.042
23	78	19	39	283.413
24	79	54	26	301.104

EXAMPLE 11

****
DESIGN OF 2-CENTRED SPIRALED COMPOUND CURVE WITH 3 SPIRALS
****

INPUT:

X--Coordinate of PI = 1000 m

Y--Coordinate of PI = 1000 m

Deflection angle at PI = 60 00 00

Azimuth of back tangent = 45 00 00

Azimuth of forward tangent = 105 00 00

R_1 = 300 m

R_2 = 500m

Length of entry spiral = 238 m

Length of exit spiral = 143 m

Δ_1 = 35 00 00

OUTPUT:

Δ_2 = 25 00 00

Length of connecting spiral = 100 m

Length of back tangent PI-TS1 = 317.075 m

Length of forward tangent PI-S3T = 320.623 m

Length of common tangent NJ = 208.281 m

Length of left circular arc = 14.259 m

Length of right circular arc = 96.666 m

Table (11-a): Computer run showing coordinates for group of points on 3-Centered Compound with 3 Spirals.

POINT NO	X--COORDINATE	Y-COORDINATE
TS1	**775.794**	**775.794**
1	786.406	786.395
2	797.051	796.962
3	807.763	807.462
4	818.574	817.861
5	829.515	828.122
6	840.616	838.210
7	851.906	848.086
8	863.410	857.710
9	875.154	867.041
10	887.158	876.036
11	899.438	884.648
12	912.010	892.828
13	924.882	900.528
14	938.058	907.694
15	951.538	914.272
S1C1	**963.459**	**919.453**
C1S2	**976.779**	**924.542**
18	983.383	926.865
19	990.006	929.147
20	996.653	931.391
21	1003.328	933.598
22	1010.035	935.768

23	1016.779	937.000
24	1023.565	939.996
25	1024.538	940.292
26	1036.113	943.500
27	1047.837	946.084
28	1059.679	948.037
29	1071.605	949.356
S2C2	**1073.599**	**949.514**
31	1098.566	950.752
32	1123.563	950.740
33	1148.529	949.479
C2S3	**1170.091**	**947.378**
35	1181.989	945.817
36	1193.849	943.994
37	1205.671	941.936
38	1217.454	939.665
39	1229.200	937.207
40	1240.910	934.585
41	1252.588	931.824
42	1264.238	928.947
43	1275.865	925.979
44	1287.474	922.942
45	1299.072	919.861
S3T	**1309.698**	**917.017**

Table (11-b): Computer run showing the length and azimuth of the line joining TS_1 to any point on curve

POINT NO	AZIMUTH			DISTANCE
1	45	01	48	15.000
2	45	07	13	30.000
3	45	16	15	45.000
4	45	28	53	59.995
5	45	45	08	74.995
6	46	04	59	89.982
7	46	28	27	104.972
8	46	55	32	119.946
9	47	26	13	134.902
10	48	00	30	149.835
11	48	38	24	164.734
12	49	19	53	179.589
13	50	04	57	194.386
14	50	53	35	209.111
15	51	45	48	223.746
16	52	33	56	236.340
17	53	29	41	250.042
18	53	57	18	256.741
19	54	24	04	263.447
20	54	50	05	270.166
21	55	15	25	276.901
22	55	40	08	283.656
23	56	04	19	290.435
24	56	28	00	297.242

25	56	31	21	298.217
26	57	12	32	309.664
27	57	57	17	320.946
28	58	45	11	332.052
29	59	35	54	342.970
30	59	44	36	344.770
31	61	32	24	367.140
32	63	17	42	389.294
33	65	00	56	411.215
34	66	28	58	430.013
35	67	17	13	440.343
36	68	04	59	450.624
37	68	52	08	460.866
38	69	38	36	471.082
39	70	24	15	481.281
40	71	09	00	491.475
41	71	52	45	501.675
42	72	35	27	511.892
43	73	17	00	522.137
44	73	57	21	532.419
45	74	36	24	542.748
46	75	11	02	552.266

EXAMPLE 12

****
DESIGN OF SIMPLE REVERSE CURVE
****

INPUT:

X--Coordinate of PI 1 = 1000

Y--Coordinate of PI 1 = 1000

Deflection angle at PI 1 = 50 00 00

Azimuth of back tangent at PI 1 = 85 00 00

Azimuth of forward tangent at PI 1 = 135 00 00

X--Coordinate of PI 2 = 1200

Y--Coordinate of PI 2 = 800

Azimuth of forward tangent at PI 2 = 80 00 00

Deflection angle at PI 2 = 55 00 00

Radius of left circular arc R_1 = 200 m

OUTPUT:

Radius of right circular arc = 364.182

Length of back tangent = 93.262

Length of common tangent = 282.843

Length of forward tangent = 189.581

Length of left circular arc = 174.533

Length of right circular arc = 349.589

Coord. of center point of left arc = (1106.332,1009.303)

Coord. of center point of right arc = (1323.461,676.538)

Table (12-a): Computer run showing coordinates for group of points on Simple Reverse curve.

POINT NO	X--COORDINATE	Y-COORDINATE
PC	**907.093**	**991.872**
1	917.073	992.194
2	927.071	992.617
3	937.073	992.239
4	947.024	991.363
5	956.928	989.999
6	966.751	988.124
7	976.469	985.770
8	986.057	982.932
9	995.491	979.619
10	1004.748	975.838
11	1013.804	971.600
12	1022.637	966.914
13	1031.225	961.793
14	1039.548	956.249
15	1047.579	950.295
16	1055.305	944.948
17	1062.705	931.223
PR	**1065.946**	**934.054**
19	1078.983	921.646
20	1092.617	909.897
21	1106.818	898.836
22	1124.543	888.490
23	1136.763	818.884

24	1152.440	870.042
25	1168.534	861.985
26	1185.007	854.733
27	1201.817	848.304
28	1218.925	842.713
29	1236.298	837.975
30	1253.864	834.100
31	1271.610	831.098
32	1289.483	828.976
33	1307.439	827.740
34	1325.434	827.393
35	1343.424	827.935
36	1361.365	829.366
37	1379.214	831.680
PT	**1386.701**	**832.920**

Table (12-b): Computer run showing the length and azimuth of the line joining PC to any point on curve.

POINT NO	AZIMUTH	DISTANCE
1	86 25 56	9.999
2	87 51 52	19.992
3	89 17 50	29.972
4	90 43 46	39.933
5	92 09 43	49.870
6	93 35 39	59.775

7	95	01	36	69.643
8	96	27	32	79.468
9	97	53	29	89.243
10	99	19	26	98.962
11	100	45	22	108.619
12	102	11	19	118.208
13	103	37	16	127.724
14	105	03	12	137.159
15	106	29	09	146.509
16	107	55	05	155.767
17	109	21	02	164.928
18	110	00	00	169.047
19	112	13	21	185.682
20	113	50	18	202.828
21	114	58	38	220.328
22	115	44	16	238.068
23	116	11	42	255.958
24	116	24	26	273.929
25	116	25	07	291.928
26	116	15	51	309.907
27	115	58	18	327.832
28	115	33	47	345.669
29	115	03	21	363.392
30	114	27	51	380.975
31	113	48	01	398.398
32	113	04	25	415.640
33	112	17	32	432.684
34	111	27	47	449.513
35	110	35	31	466.111

36	109	41	01	482.463
37	108	44	31	498.557
38	108	20	10	505.261

EXAMPLE 13

****
DESIGN OF SPIRALED REVERSE CURVE
****

INPUT:

X--Coordinate of PI 1 = 1000

Y--Coordinate of PI 1 = 1000

Deflection angle at PI 1 = 50 00 00

Azimuth of back tangent at PI 1 = 85 00 00

Azimuth of forward tangent at PI 1 = 135 00 00

X--Coordinate of PI 2 = 1300

Y--Coordinate of PI 2 = 700

Azimuth of forward tangent at PI 2 = 80 00 00

Deflection angle at PI 2 = 55 00 00

Radius of left circular arc R_1 = 200 m

Radius of right circular arc R_2 = 400 m

Length of spiral 1, L_{s1} = 150 m

Length of spiral 2, L_{s2} = 100 m

Length of spiral 3, L_{s3} = 150 m

OUTPUT:

Length of spiral 4, L_{s4} = 193.018 m

Length of left circular arc = 49.533 m

Length of right circular arc = 212.464 m

Back tangent of left part of reverse curve = 166.712 m

Forward tangent of left part of reverse curve = 147.502 m

Back tangent of right part of reverse curve = 276.762 m

Forward tangent of right part of reverse curve = 314.164 m

Table (13-a): Computer run showing coordinates for group of points on Spiraled Reverse curve.

Point No	X--Coordinate	Y--Coordinate
TS1	**833.923**	**985.470**
1	838.904	985.905
2	843.885	986.336
3	848.868	986.759
4	853.851	987.169
5	858.835	987.653
6	863.821	987.935
7	868.809	988.283
8	873.799	988.602
9	878.791	988.887
10	883.785	989.135
11	888.780	989.342
12	893.778	989.502
13	898.777	989.613
14	903.776	989.670
15	908.776	989.668
16	913.776	989.603
17	918.774	989.472
18	923.770	989.271
19	928.762	988.994
20	933.749	988.638
21	938.730	988.199
22	943.702	987.673
23	948.664	987.056
24	953.613	986.345

25	958.547	985.534
26	963.462	984.620
27	968.357	983.601
28	973.228	982.471
29	978.071	981.228
30	982.882	979.869
S1C1	**982.882**	**979.869**
32	992.396	976.793
33	1001.745	973.245
34	1010.904	969.234
35	1019.852	964.771
C1S2	**1028.163**	**960.104**
37	1032.432	957.501
38	1036.637	954.797
39	1040.781	951.998
40	1044.864	949.113
41	1048.889	946.146
42	1052.857	943.105
43	1056.772	939.994
44	1060.636	936.821
45	1064.451	933.589
46	1068.222	930.306
47	1071.951	926.975
48	1075.643	923.603
49	1079.301	920.194
50	1082.928	916.754
51	1086.530	913.286
52	1090.110	909.795
53	1093.673	906.287

54	1097.223	902.766
55	1100.763	899.235
SS	**1104.300**	**895.701**
57	1111.373	888.631
58	1118.457	881.574
59	1125.566	874.540
60	1132.709	867.542
61	1139.899	860.592
62	1147.146	853.702
63	1154.462	846.885
64	1161.857	840.154
65	1169.342	833.521
66	1176.924	827.002
67	1184.615	820.610
68	1192.421	814.360
69	1200.351	808.268
70	1208.410	802.349
71	1216.606	796.619
S3C2	**1216.606**	**796.619**
73	1233.411	785.779
74	1250.737	775.792
75	1268.540	766.684
76	1286.776	758.477
77	1305.399	751.191
78	1324.364	744.845
79	1343.622	739.455
80	1363.125	735.035
81	1382.825	731.594
82	1402.672	729.143

C2S4	**1415.092**	**728.118**
84	1427.710	735.911
85	1437.705	735.604
86	1447.705	735.520
87	1457.704	735.648
88	1467.698	735.974
89	1477.685	736.485
90	1487.661	737.168
91	1497.626	738.010
92	1507.577	738.997
93	1517.514	740.117
94	1527.437	741.357
95	1537.346	742.704
96	1547.241	744.144
97	1557.125	745.665
98	1566.998	747.253
99	1576.862	748.897
100	1586.719	750.583
101	1596.570	752.298
102	1606.419	754.030
S4T	**1609.392**	**754.554**

Table (13-b): Computer run showing the length and azimuth of the line joining TS_1 to any point on curve

Point No	Azimuth			Distance
1	85	01	54	10.000
2	85	04	18	15.000
3	85	07	38	20.000
4	85	11	56	25.000
5	85	17	11	40.000
6	85	23	23	34.999
7	85	30	33	39.999
8	85	38	40	44.998
9	85	47	44	49.996
10	85	57	46	54.994
11	86	08	45	59.990
12	86	20	41	64.986
13	86	33	34	69.979
14	86	47	25	74.971
15	87	02	13	79.960
16	87	17	58	84.945
17	87	34	40	89.927
18	87	52	19	94.904
19	88	10	56	99.877
20	88	30	30	104.843
21	88	51	00	109.801
22	89	12	28	114.752
23	89	34	53	119.693
24	89	58	14	124.624
25	90	22	32	129.542

26	90	47	47	134.447
27	91	13	59	139.337
28	91	41	07	144.210
29	92	09	12	149.065
30	92	09	12	149.065
31	93	08	03	158.711
32	94	09	59	168.266
33	95	14	29	177.724
34	96	21	09	187.077
35	97	26	24	195.889
36	98	01	12	200.469
37	98	36	15	205.022
38	99	11	28	209.548
39	99	46	45	214.051
40	100	21	59	218.533
41	100	57	06	222.996
42	101	32	01	227.442
43	102	06	40	231.874
44	102	40	59	236.294
45	103	14	55	240.705
46	103	48	24	245.110
47	104	21	22	249.511
48	104	53	48	253.912
49	105	25	39	258.313
50	105	56	51	262.719
51	106	27	23	267.130
52	106	57	12	271.551
53	107	26	16	275.983
54	107	54	33	280.429

55	108	22	01	284.890
56	109	14	26	293.864
57	110	03	33	302.910
58	110	49	29	312.027
59	111	32	19	321.216
60	112	12	06	330.478
61	112	48	56	339.811
62	113	22	52	349.215
63	113	53	57	358.689
64	114	22	16	368.231
65	114	47	49	377.839
66	115	10	41	387.509
67	115	30	53	397.240
68	115	48	29	407.025
69	116	03	29	416.862
70	116	15	57	426.744
71	116	15	57	426.744
72	116	33	32	446.617
73	116	42	16	466.581
74	116	43	14	486.579
75	116	37	20	506.559
76	116	25	22	526.476
77	116	08	02	546.290
78	115	45	54	565.965
79	115	19	30	585.468
80	114	49	16	604.769
81	114	15	37	623.842
82	113	53	04	635.600
83	112	47	46	644.098

84	112	28	53	653.442
85	112	09	27	662.724
86	111	49	33	671.947
87	111	29	16	681.116
88	111	08	41	690.234
89	110	47	51	699.305
90	110	26	52	708.335
91	110	05	46	717.327
92	109	44	38	726.288
93	109	23	30	735.223
94	109	02	26	744.136
95	108	41	29	753.035
96	108	20	41	761.923
97	108	00	06	770.809
98	107	39	46	779.695
99	107	19	44	788.589
100	107	00	01	797.496
101	106	40	41	806.421
102	106	34	56	809.119

ملخص

حساب وتوقيع عناصر منحنيات الوصل اللولبية باستخدام اجهزة الكترونية وبرامج حاسوبية

المهندس ماهر طعيمة داود

تم في هذا البحث تطوير نماذج رياضية متنوعة بهدف تصميم وتوقيع المنحنيات الدائرية البسيطة والمركبة والعكسية والانتقالية، ولقد تم التركيز في هذه الرسالة على مختلف انواع المنحنيات الانتقالية (اللولبية).

باستخدام هذه النماذج الرياضية صمم برنامج حاسوبي ليقوم بالمهمات التالية:

١- القيام بعملية ضبط للاحداثيات والاتجاهات للخط الواصل بين سلسلة من نقاط التقاطع.
٢- حساب مختلف عناصر اي منحنى.
٣- حساب الاحداثيات المطلقة لمجموعة من النقاط على اي منحى.
٤- حساب مقدار الاتجاه الدائري الكلي وطول المسافة للخط الواصل بين نقطة بداية اي منحنى واية نقطة اخرى على المنحنى.

من المتوقع ان يساعد هذا البحث في توقيع المنحنيات المختلفة بطريقة دقيقة وسريعة وفعالة.

الجامعة الاردنيــــة
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رسالة الماجستير المقدمه مــــن قبـــل

المهندس ماهر طعيمة داود

المشرف :

الاستاذ الدكتور يوسف صيام

تشرين أول ١٩٩٣

عمـــــان - الاردن